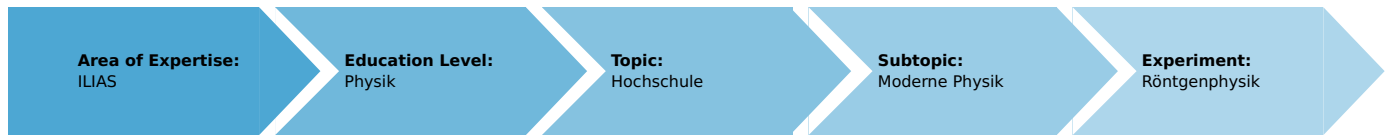


## Determination of the Rydberg constant, Moseley's law, and screening constant (Item No.: P2541001)

### Curricular Relevance



#### Difficulty



Difficult

#### Preparation Time



1 Hour

#### Execution Time



2 Hours

#### Recommended Group Size



2 Students

#### Additional Requirements:

- PC

#### Experiment Variations:

#### Keywords:

Characteristic X-radiation, Bohr model, energy levels, binding energy, Moseley's law, Rydberg frequency, screening constant, Bragg scattering

### Overview

### Short description

#### Principle

Moseley's law describes the relationship between the energy of the  $K_{\alpha}$  lines of characteristic X-ray spectra and the atomic number. In this experiment, the characteristic X-ray lines of various different anode materials are determined in order to verify Moseley's law.

This experiment is included in the "XRC 4.0 X-ray characteristics" upgrade set.



# Student's Sheet

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## Equipment

Position No.	Material	Order No.	Quantity
1	XR 4.0 expert unit, X-ray unit, 35 kV	09057-99	1
2	XR 4.0 X-ray goniometer	09057-10	1
3	XR 4.0 X-ray Plug-in Cu tube	09057-51	1
4	XR 4.0 X-ray Plug-in Mo tube	09057-61	1
5	XR 4.0 X-ray Plug-in Fe tube	09057-71	1
6	Geiger-Mueller counter tube, 15 mm (type B)	09005-00	1
7	XR 4.0 X-ray LiF crystal, mounted	09056-05	1
8	XR 4.0 Software measure X-ray	14414-61	1
9	Data cable USB, plug type A/B, 1.8 m	14608-00	1
10	XR 4.0 X-ray Diaphragm tube $d = 2$ mm	09057-02	1

## Tasks

1. Record the X-ray spectra of the three X-ray tubes.
2. Determine the wavelengths and frequencies of the characteristic X-ray lines based on the Bragg angles of the lines.
3. Create the Moseley lines and determine the Rydberg constant and screening constant.

## Set-up and procedure

### Set-up

Connect the goniometer and the Geiger-Müller counter tube to their respective sockets in the experiment chamber (see the red markings in Fig. 2). The goniometer block with the analyser crystal should be located at the end position on the right-hand side.

Fasten the Geiger-Müller counter tube with its holder to the back stop of the guide rails. Do not forget to install the diaphragm in front of the counter tube (see Fig. 3).

Insert a diaphragm tube with a diameter of  $2\text{ mm}$  into the beam outlet of the tube plug-in unit for the collimation of the X-ray beam.

**For calibration:** Make sure, that the correct crystal is entered in the goniometer parameters. Then, select "Menu", "Goniometer", "Autocalibration". The device now determines the optimal positions of the crystal and the goniometer to each other and then the positions of the peaks



Fig. 2: Connectors in the experiment chamber

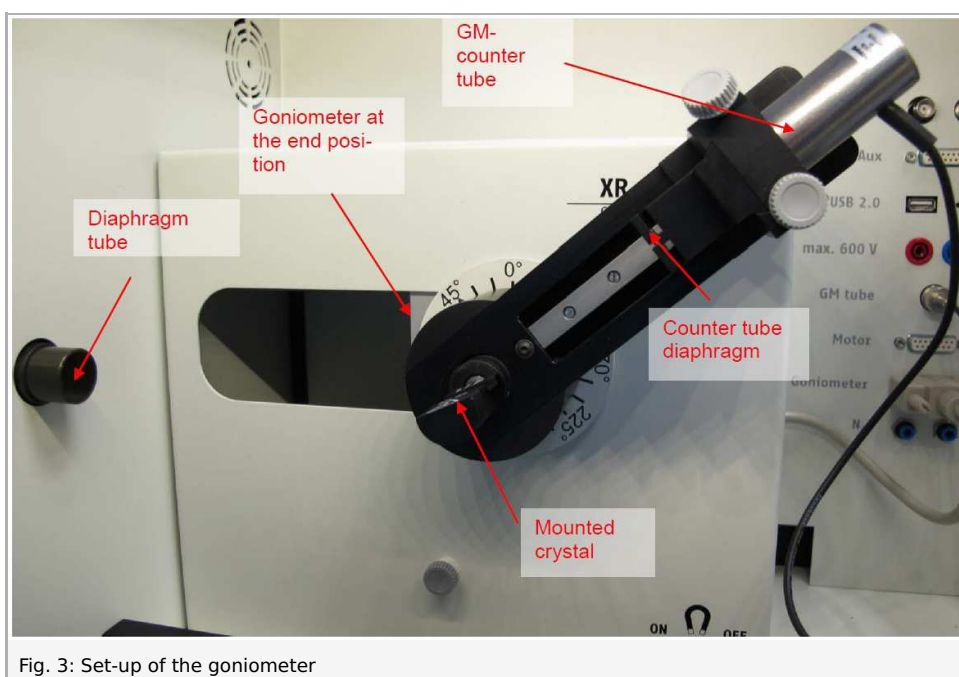


Fig. 3: Set-up of the goniometer

## Note

Details concerning the operation of the X-ray unit and goniometer as well as information on how to handle the monocrystals can be found in the respective operating instructions.

## Procedure

- Connect the X-ray unit via USB cable to the USB port of your computer (the correct port of the X-ray unit is marked in Fig. 4).

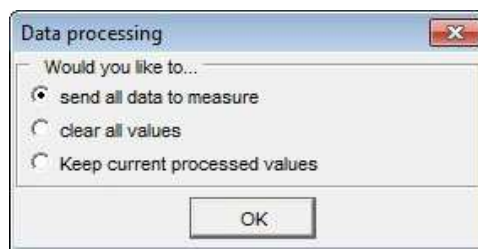


Fig. 4: Connection of the computer

- Start the "measure" program. A virtual X-ray unit will be displayed on the screen (Fig. 5).
- You can control the X-ray unit by clicking the various features on and under the virtual X-ray unit. Alternatively, you can also change the parameters at the real X-ray unit. The program will automatically adopt the settings.
- Click the experiment chamber (see the red marking in Figure 5) to change the parameters for the experiment. Start angle:  $3^\circ - 4^\circ$ . Record the spectra at least up to the second-order characteristic lines.
- If you click the X-ray tube (see the red marking in Figure 5), you can change the voltage and current of the X-ray tube. Select the settings as shown in Figure 6.
- Start the measurement by clicking the red circle:



- After the measurement, the following window appears:



- Select the first item and confirm by clicking OK. The measured values will now be transferred directly to the "measure" software.
- At the end of this manual a short introduction to the evaluation of data using the program "measure" is given.

## Note

Never expose the Geiger-Müller counter tube to the primary X-radiation for an extended period of time.



Fig. 5: Part of the user interface of the software

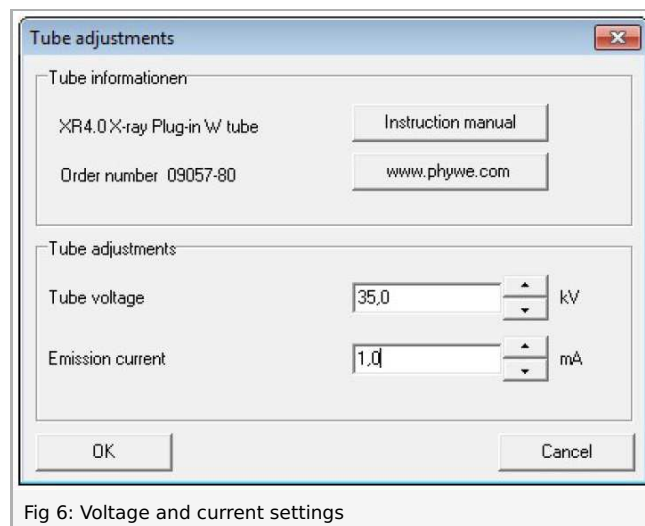


Fig 6: Voltage and current settings

### Overview of the settings of the goniometer and X-ray unit:

- 2:1 coupling mode
- Gate time 2 s; angle step width  $0.1^\circ$
- Start angle:  $3^\circ - 4^\circ$ . Record the spectra at least up the second-order characteristic lines.
- Anode voltage  $U_A = 35 \text{ kV}$ ; anode current  $I_A = 1 \text{ mA}$

## Theory and evaluation

### Theory

H. G. J. Moseley discovered the relationship between the energy of the  $K_\alpha$  lines of characteristic X-ray spectra and the atomic number. If the root of the frequency of the  $K_\beta$  line is plotted as a function of the atomic number  $Z$  of the anode material, a straight line results.

Based on this straight line, the order of the elements in the periodic table of elements was specified in a definite manner for the very first time. The element hafnium (**Hf**) ( $Z = 72$ ) that had been unknown hitherto, was represented as a gap on Moseley's straight line. Following the discovery of hafnium and the recording of the X-ray spectrum, the element fitted right into this gap, which substantiated Moseley's findings.

The following is valid for the binding energy  $E_n$  of an electron on a shell with the principal quantum number  $n$ :

$$E_n = -\frac{m_e e^4}{8\varepsilon_0^2 h^2} (Z - \sigma)^2 \quad \text{where } (n = 1, 2, 3, \dots) \quad (1)$$

Electron mass	$m_e$	$= 9.1091 \cdot 10^{-31} \text{ kg}$
Elementary charge	$e$	$= 1.6021 \cdot 10^{-19} \text{ As}$
Planck's constant	$h$	$= 6.6256 \cdot 10^{-34} \text{ Js}$
Dielectric constant	$\varepsilon_0$	$= 8.8544 \cdot 10^{-12} \text{ N}^{-1} \text{ m}^{-2} \text{ C}^2$
Atomic number	$Z$	
Screening constant	$\sigma$	

During the transition of an electron from  $L$  shell to a free space on the  $K$  shell of an atom, the energy that is released can be converted into X-radiation. The frequency  $f$  of this quantum can be determined with the aid of equation (1):

$$f = \frac{\Delta E}{h} = \frac{m_e e^4}{8\varepsilon_0^2 h^3} (Z - \sigma)^2 \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad \text{Moseley's law} \quad (2)$$

$$f_R \cdot (Z - \sigma)^2 \left( \frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$(f_R = \frac{m_e e^4}{8\varepsilon_0^2 h^3} = 3.2899 \cdot 10^{15} \text{ s}^{-1} = \text{Rydberg frequency})$$

With  $n_1 = 1$  and  $n_2 = 2$ , it follows from (2) that:

$$\sqrt{f} = \frac{1}{2} \sqrt{3 f_R} (Z - \sigma) \quad (3)$$

If the interplanar spacing  $d$  of the analyser crystal is known, the glancing angles  $\vartheta$  of the characteristic  $K_\alpha$  and  $K_\beta$  lines can be used to determine the wavelengths  $\lambda$  of the lines based on Bragg's law.

$$2d \sin \vartheta = n \lambda \quad (n = 1, 2, 3, \dots) \quad (4)$$

(**LiF**(200) interplanar spacing  $d = 201.4 \text{ pm}$ )

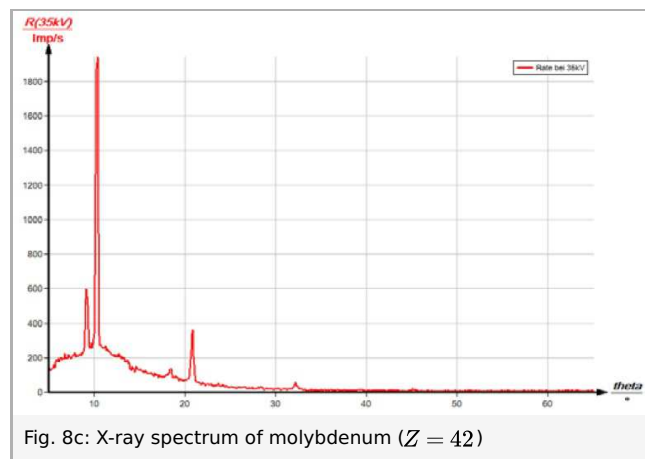
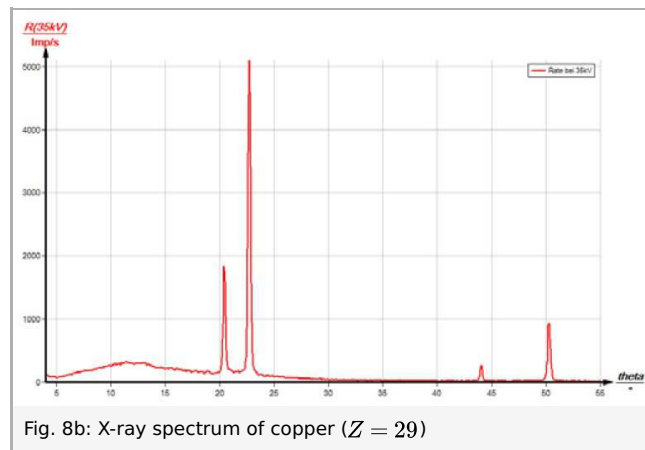
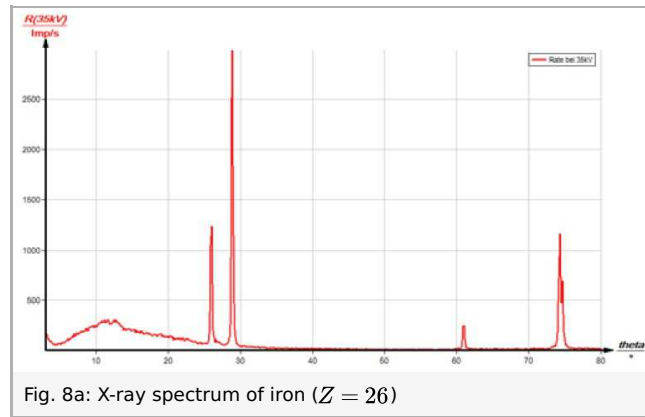
The associated frequencies  $f$  of the characteristic lines result from:

$$c = \lambda \cdot f \quad (\text{velocity of light } c = 2.9979 \cdot 10^8 \text{ m/s}).$$

## Evaluation

Task 1: Record the X-ray spectra of the three X-ray tubes.

The X-ray spectra of iron, copper, and molybdenum with the **LiF** crystal as the analyser are shown in Figure 8a-8c.



Task 2 Determine the wavelengths and frequencies of the characteristic X-ray lines based on the Bragg angles of the lines.

Table 1 shows the  $\vartheta$  values of the characteristic  $K_{\alpha}$  and  $K_{\beta}$  lines of the three anode materials that were determined based on the spectra as well as the associated wavelength and frequency values that were determined with the aid of equations (3) and (4).

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	n = 1		n = 2		n = 3		$\bar{\lambda} / \text{pm}$	$\sqrt{f(K_{\alpha/\beta})}/10^8 \text{ s}^{-1}$
	$\vartheta/^\circ$	$\lambda/\text{pm}$	$\vartheta/^\circ$	$\lambda/\text{pm}$	$\vartheta/^\circ$	$\lambda/\text{pm}$		
<b><math>K_\alpha</math> lines</b>								
Fe (Z = 26)	28.9	194.7	74.3	193.9	-	-	194.3	12.42
Cu (Z = 29)	22.6	154.1	50.2	154.9	-	-	154.5	13.93
Mo (Z = 42)	10.2	70.4	20.8	71.2	32.1	71.3	71.0	20.55
<b><math>K_\beta</math> lines</b>								
Fe (Z = 26)	25.8	175.3	60.9	176.0	-	-	175.7	13.06
Cu (Z = 29)	20.4	140.4	43.9	139.6	-	-	140.0	14.63
Mo (Z = 42)	9.2	64.4	18.5	63.9	28.2	63.4	63.9	21.66

Table 1:

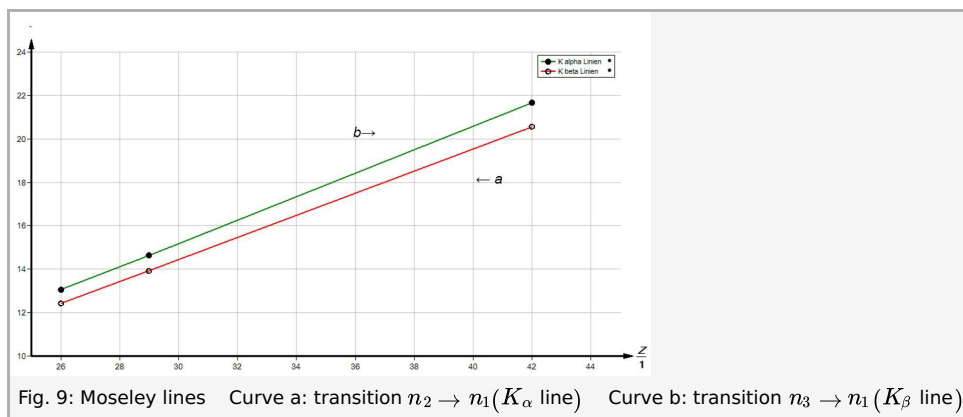
Task 3: Create the Moseley lines and determine the Rydberg constant and screening constant.

Figure 9 shows the two Moseley lines that result from the calculated values (see table 1). The mean value of the two gradients

$$m = 0.5 \cdot 10^8 \text{ s}^{-1/2} = \frac{1}{2} \sqrt{3f_R}$$

leads to the Rydberg frequency  $f_R = 3.33 \cdot 10^{15} \text{ s}^{-1}$ . The screening constant can be determined with the aid of equation (3):  $\sigma_{2,1} \approx 1$ .

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## “measure” software

We recommend performing the graphical evaluation with the “measure” software.

On the tab “Measurement”, click “Enter data manually”. Then, enter the number of measurement values into the corresponding field (in this case “3”) and enter also the number of channels (in this case “2”).

Click “Next” and enter the values into the corresponding fields (under “Number” the x-values, i.e. the atomic number and the calculated values into the channels).

The resulting straight lines nearly superimpose each other since they are scaled individually based on the left axis. In order to change this, click the button  on the top bar and select “Fit collectively”.

Right-click the spectrum if you would like to display the data table or change the display options. You can, for example, change the names of the channels or select a certain line type. If you click one of the lines and then “Analysis” -> “Show slope”, the slope of the selected straight line can be displayed as shown in Figure 10.



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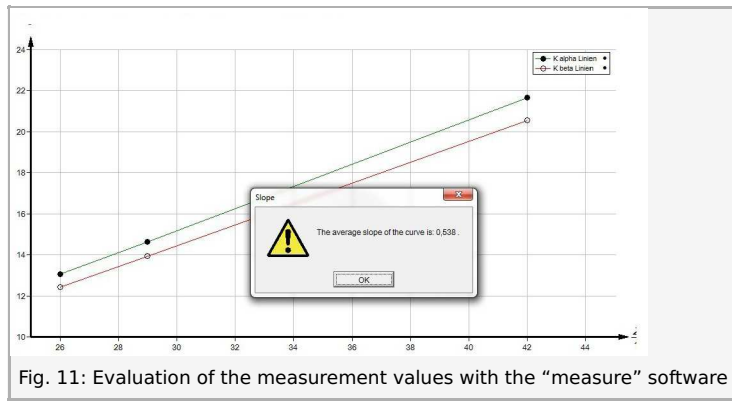


Fig. 11: Evaluation of the measurement values with the "measure" software