Chladni figures (Item No.: P2150501)

Curricular Relevance

Area of Expertise:	Education Level:	Topic:	Subtopic:	Experiment:
Physics	University	Oscillations and Mechanical Waves, Acoustics	Oscillatory Motion	Chladni figures

Difficulty	Preparation Time	Execution Time	Recommended Group Size
Difficult	1 Hour	1 Hour	2 Students

Additional Requirements:

Experiment Variations:

Keywords:

wavelength, stationary waves, eigen-modes, two-dimensional standing waves, acoustic vibrations

Overview

Short description

Square and round metal plates are brought to vibrate through acoustic stimulations by a loudspeaker. When the driving frequency corresponds to a given eigen-frequency (natural vibration mode) of the plate, the nodal lines are made visible with sand. The sand is expelled from the vibrating regions of the plate and gathers in the lines because these are the only places where the amplitude of vibrations is close to zero.

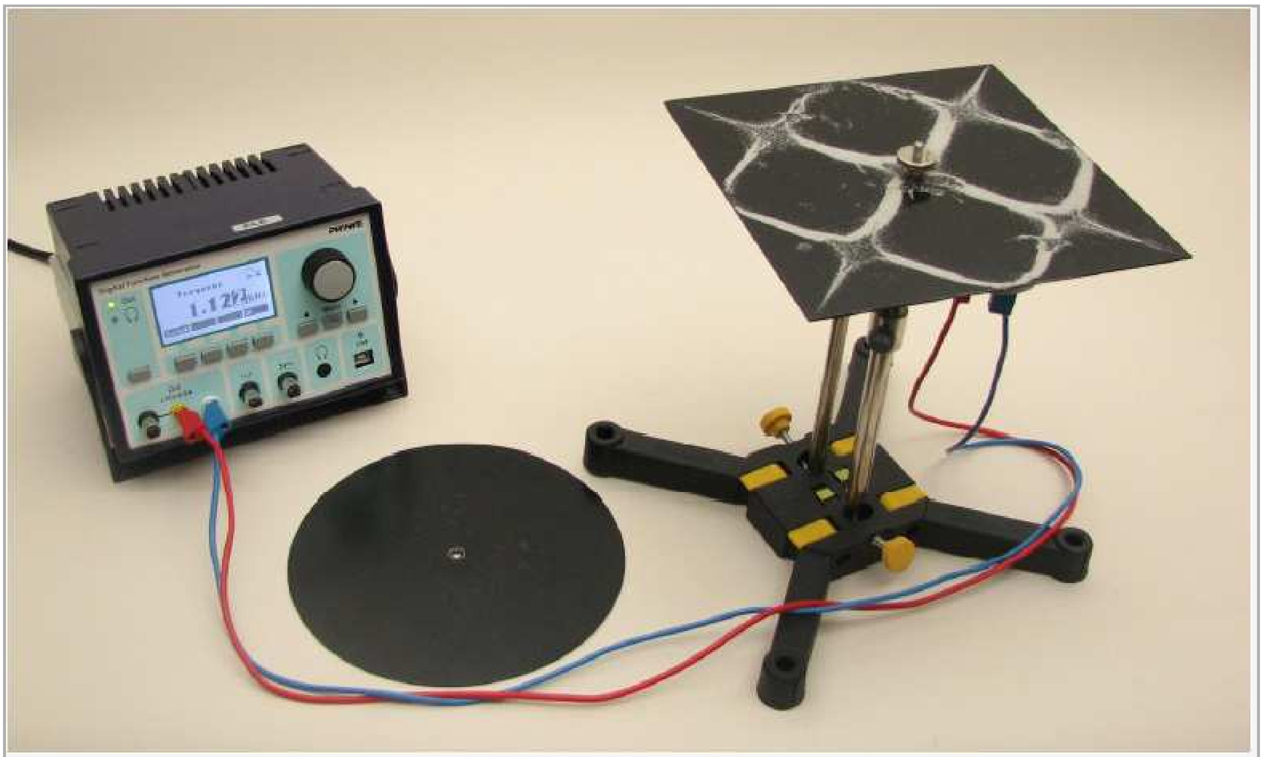

Fig. 1: Experimental setup.

Equipment

Position	Material	Order nr.	Quantity
1	PHYWE Digital Function Generator, USB	13654-99	1
2	Loudspeaker / Sound head, 8 ohms	03524-01	1
3	Sound pattern plates	03478-00	1
4	Support base, variable	02001-00	1
5	Stand tube	02060-00	1
6	Sea sand, purified 1000 g	30220-67	1
7	Boss head	02043-00	1
8	Support rod, stainless steel, l = 250 mm, d = 10 mm	02031-00	1
9	Connecting cord, 32 A, 500 mm, red	07361-01	1
10	Connecting cord, 32 A, 500 mm, blue	07361-04	1

Task

Determine the frequencies at which resonance occurs and drive the plate specifically at these frequencies.

Set-up and procedure

Set up the equipment as seen in Fig. 1. The speaker is mounted on the support rod through the boss head and is facing upwards. Make sure that the speaker is close to the plate but is not in contact with it. The only thing touching the plate should be the stand tube on which it is mounted!
Set the generator signal to sine and the frequency to about 100 Hz. Adjust the signal amplitude so that the sound from the speaker is loud but still bearable. Distribute some sand over the plate and start changing the frequency slowly from 0.1 to 2 kHz. At some frequencies patterns as seen in Fig. 2 should emerge. The specific frequencies may differ from the here given values because even smallest production variations significantly detune the modes of vibration. When you notice the sand vibrating and a figure appearing on the plate, fine-tune the frequency to obtain the best result. You may occasionally add more sand as it is removed from the plate by vibrations.

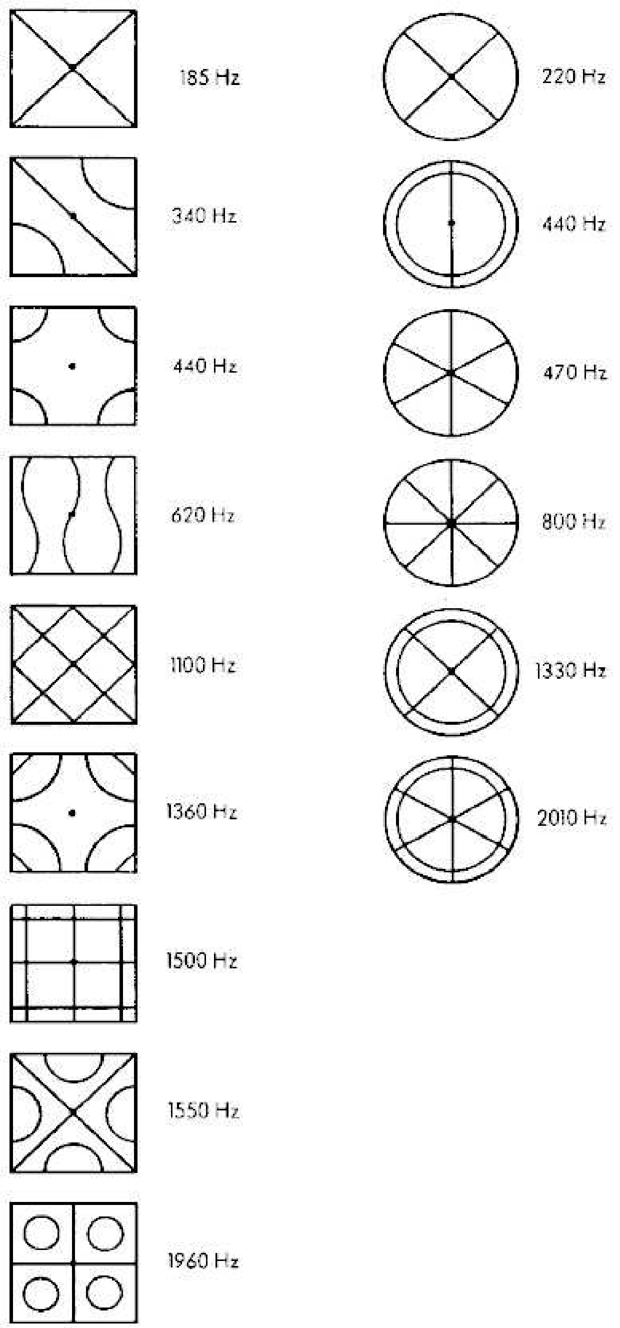

Fig. 2: Some Chladni figures with corresponding frequencies.

Theory and Evaluation

The figures seen here result from flexural vibrations. The frequency of such a natural vibration is proportional to the plate's thickness and is dependent on Young's modulus, the density, and also, because of the two-dimensionality of the waves, on the transverse contraction coefficient of the plate's material as well. Here, with a thick plate and a freely vibrating rim, for circular

plates the exact solutions are only to be found with advanced mathematics, and for rectangular plates only numerical solutions exist.
For thin plates with a fixed rim the two-dimensional wave equation

$$\partial_{xx}u(x,y,t)+\partial_{yy}u(x,y,t)=\frac{1}{c^2}\partial_{tt}u(x,y,t) \quad (1)$$

yields solutions to the problem.

The solutions to Eq. (1) for a rectangle of width a and length b and boundary condition $u=0$ for $x=0$ and $y=0$ are standing waves

$$u(x,y,t)=A\sin\left(\frac{n\pi}{a}x\right)\sin\left(\frac{m\pi}{b}y\right)\cos(\omega t) \quad (2)$$

with frequencies

$$\omega=\sqrt{\left(\frac{n}{a}\right)^2+\left(\frac{m}{b}\right)^2} \quad (3)$$

The solutions to Eq. (1) for a rectangle of width a and length b and boundary condition $u=0$ for $x=0$ and $y=0$ are standing waves

$$u(r,\Theta,t)=A\cdot J_n(kr)\cos(n\Theta)\sin(\omega t) \quad (4)$$

where J_n is the n^{th} order Bessel function and k is such that $a_k=z_n,m$; z_n,m being the m^{th} root of the n^{th} order Bessel function (value where J_n is zero). A zero of the Bessel function must occur at the boundary $r=a$ and zeros occurring before the m^{th} zero form ($m-1$) concentric circular nodes.

The principle of standing waves can be illustrated by an example of a tube (Fig. 3). Notice the different number of nodes as the eigen-frequencies increase, and how the form of the standing waves depends on the boundary conditions (closed tube in the left panel as opposed to open tube in the right one).

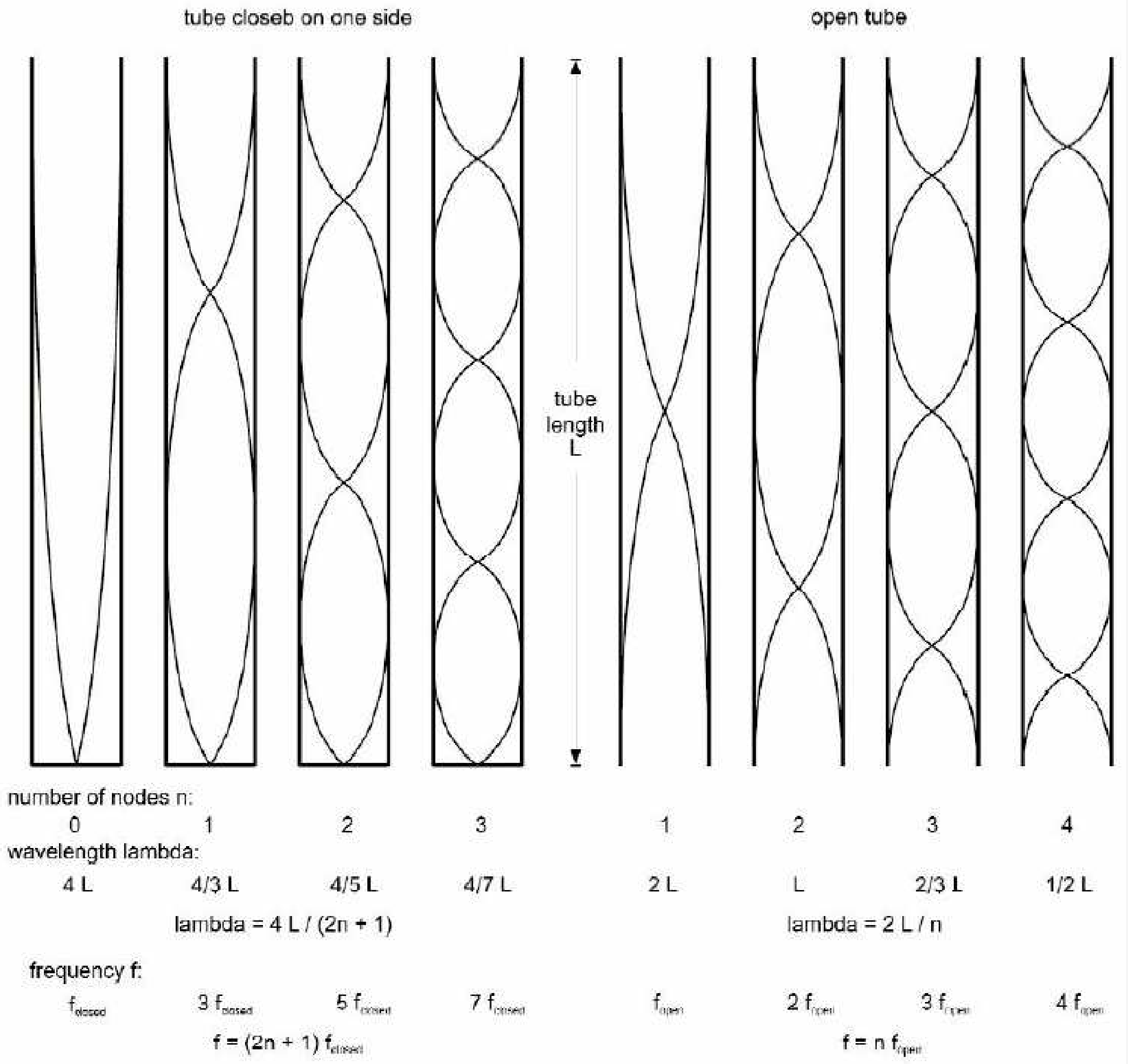

Fig. 3: Schematic illustration of standing waves in a tube.