Compton effect – energy-dispersive direct measurement (Item No.: P2546001)

Curricular Relevance

| Area of Expertise: ILIAS | Education Level: Physik | Topic: Hochschule | Subtopic: Moderne Physik | Experiment: Röntgenphysik |

Difficulty	Preparation Time	Execution Time	Recommended Group Size
Difficult	1 Hour	2 Hours	2 Students

Additional Requirements:

- PC

Experiment Variations:

Keywords:

Bremsstrahlung, characteristic X-radiation, Compton scattering, Compton wavelength, conservation of energy and momentum, rest mass and rest energy of the electron, relativistic electron mass and energy, semiconductor detector and multichannel analyser

Overview

Short description

Principle

Photons of the molybdenum K_α X-ray line are scattered at the quasi-free electrons of an acrylic glass cuboid. The energy of the scattered photons is determined in an angle-dependent manner with the aid of a swivelling semiconductor detector and a multi-channel analyser.

Fig. 1: Experimental set-up

Equipment

Position No.	Material	Order No.	Quantity
1	XR 4.0 expert unit, X-ray unit, 35 kV	09057-99	1
2	XR 4.0 X-ray goniometer	09057-10	1
3	XR 4.0 X-ray Plug-in Mo tube	09057-61	1
4	Multichannel analyser	13727-99	1
5	XR 4.0 X-ray energy detector (XRED)	09058-30	1
6	XR 4.0 X-ray Compton attachment	09057-04	1
7	measure Software multi channel analyser	14452-61	1
8	XR 4.0 X-ray Diaphragm tube d = 1 mm	09057-01	1
9	XR 4.0 X-ray Diaphragm tube d = 5 mm	09057-03	1

Tasks

1. Calibrate the semiconductor energy detector.
2. Energy determination of the photons of the W-L_α-line that are scattered through an acrylic glass element as a function of the scattering angle.
3. Compare the measured energy values of the lines of scatter with the calculated energy values.
4. Calculate the Compton wavelength of electrons and a comparison of this value with the corresponding value of the $90°$ scatter-ing.

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Set-up and procedure

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Set-up (Fig. 1)

- Screw the adapter ring onto the inlet tube of the energy detector.

- Connect the signal and supply cables to the corresponding ports of the detector with the aid of the right-angle plugs.

- Connect the signal and supply cables from the MCA to the appropriate connections in the experiment chamber of the X-ray unit (signal cable: red, supply cable: green (see Fig. 2)).

Fig. 2: Connections in the experimentation area

- Connect the external ports for the X RED of the x-ray unit (signal cable red, supply cable green, see Fig. 3) to the multi-channel analyse (MCA). Connect the signal cable via a screened BNC-cable to the "Input" port of the MCA and the supply cable to the "X-Ray Energy Det." port of the MCA.

- Secure the energy detector in the holder of the swivel arm of the goniometer. Lay the two cables with sufficient length so that the goniometer can be swivelled freely over the entire swivelling range.

- Connect the multi-channel analyser and computer with the aid of the USB cable.

- Insert the tube with the 2-mm-aperture.

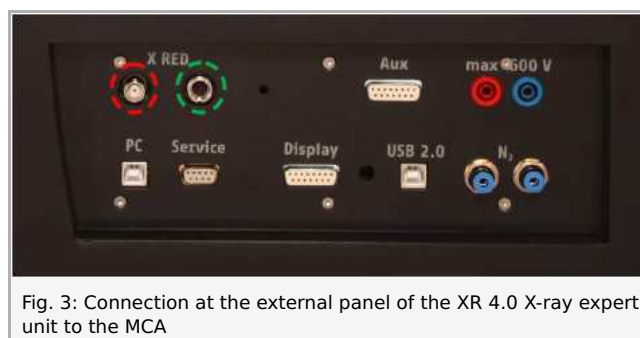- Bring the goniometer block and the detector to their respective end positions on the right.

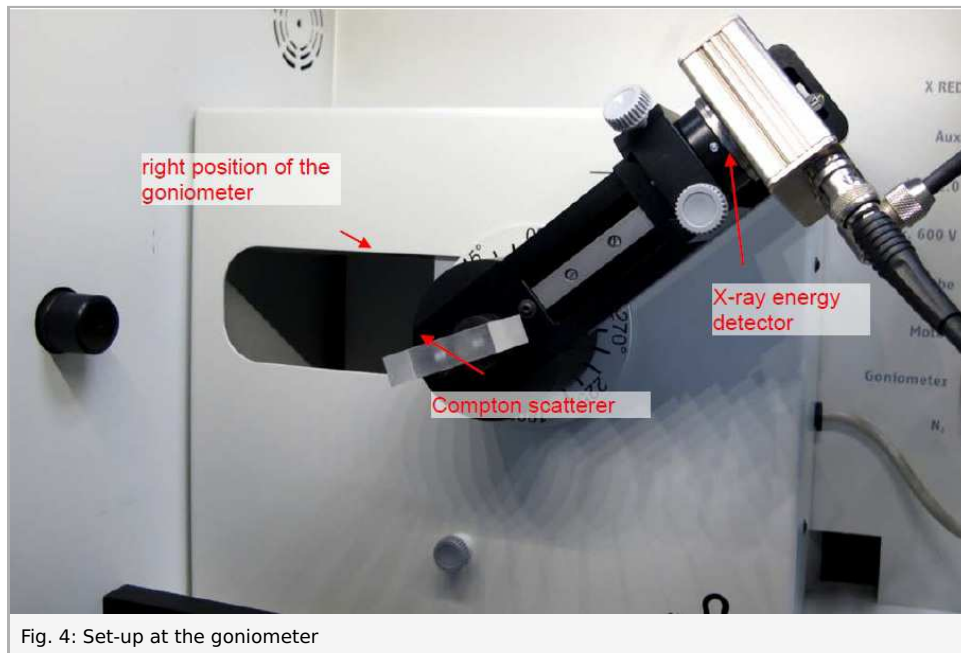Fig. 3: Connection at the external panel of the XR 4.0 X-ray expert unit to the MCA

Fig. 4: Set-up at the goniometer

Calibration of the multi-channel analyser

(if there is no other already existing calibration that can be used)

- Bring the goniometer block and the detector to their respective end positions on the right.

- Insert the tube with the 1mm-aperture into the exit tube of the X-ray tube.

- With the X-ray unit switched on and the door locked, bring the detector to the $0°$ position. Then, shift the detector by some tenths degree out of the zero position in order to reduce the total rate.

- Operating data of the tungsten X-ray tube: Select an anode voltage $U_A = 25\,\mathrm{kV}$ and an anode current $I_A = 0.02\,\mathrm{mA}$ and confirm these values by pressing the "Enter" button.

- Switch on the X-radiation.

- In the MEASURE program, select "Multi channel analyser" under "Gauge". Then, select "Settings and calibration". After the "Calibrate" button has been clicked, a spectrum can be measured. The count-ing rate should be $< 300\,\mathrm{c/s}$. Energy calibration settings: - 2-point calibration, - Unit = keV, Gain = 2 – Set the offset so that low-energy noise signals will be suppressed (usually a few per cent are sufficient), See Fig. 5.

- Measuring time: 5 minutes. Use the timer of the X-ray unit.

- Make the two coloured calibration lines congruent with the line centres of the two characteristic X-ray lines. The corresponding energy values (see e.g. P2544701) $E(\mathrm{L_3\,M_5/L_3\,M_4}) = 8,41\,\mathrm{keV}$ and $E(\mathrm{L_2\,N_4}) = 9,69\,\mathrm{keV}$ are entered into the corresponding fields, depending on the colour. (Note: Since a separa-tion of the lines $\mathrm{L_3\,M_5}$ and $\mathrm{L_3\,M_4}$ Lines is not possible, the mean value of both lines is entered as the energy of the line).
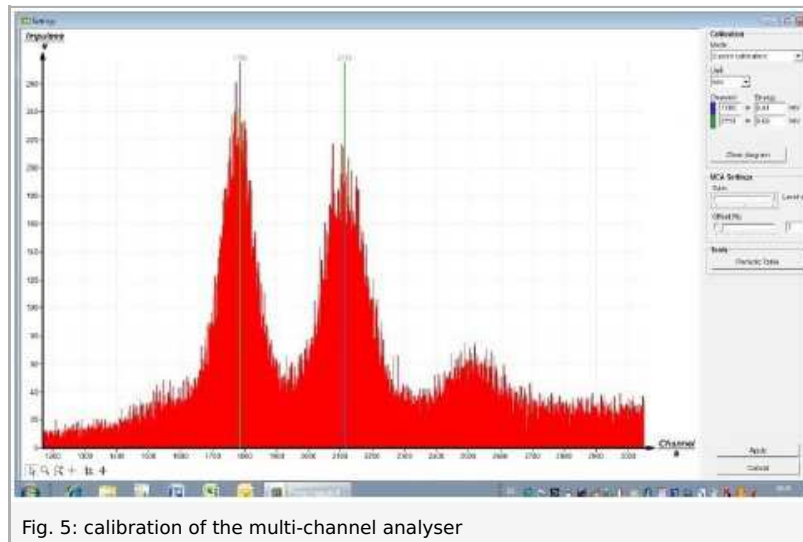
- Name and save the calibration.

Fig. 5: calibration of the multi-channel analyser

Compton scattering

Set the detector to the zero position and select the following operating data: diaphragm tube with $d = 1\,\text{mm}$, $U_A = 30\,\text{kV}$, $I_A = 0,08\,\text{mA}$.

- Enter the following parameters into the field "Control" in the window "Spectra recording": - Gain = 2, – Offset = $5\,\%$, - X-Data = keV, - Interval width [channels] = 1.

- Start the X-ray tube. The measuring time should be approximately 5 minute so that the intensity of the K_α-peak is approximately 200-300 pulses. Accept the data and save them.

- Place the acrylic glass element (scatterer) of the Compton equipment into the sample holder and set it to a 10° position. Set the detector to 20°.

- Now, add the tube with the 5 mm aperture and increase the operating data of the X-ray tube to $U_A = 30\,\text{kV}$ and $I_A = 0.3\,\text{mA}$.

- Start the measurement. The measuring time is approximately 10 minutes. The intensity of the K_α-peak should be approximately 200 pulses. Stop the measurement with "Accept data".

- Leave the acrylic glass scatterer in its position and perform additional measurements. To do so, change the angle of the detector in steps of 10° up to the final value of 160°.

Evaluation of the measurement curves

- In order to determine the line energy, switch from the bar display to the curve display. To do so, click "Display options" and then "Interpolation and straight lines".

- Extend the relevant line section with the aid of the zoom function 🔍.

- Then, select the curve section with ➕ . Open the window "Function fitting 📊 ". Then, select "Scaled normal distribution".

- Find the line centroid of the normal distribution with "Peak analysis 📈 " or determine it with the function "Survey ▦ ".

Theory and evaluation

Theory and Evaluation

Figure 6 shows a schematic representation of the Compton Effect.

Fig. 6: Momentum and energy relation of the Compton effect
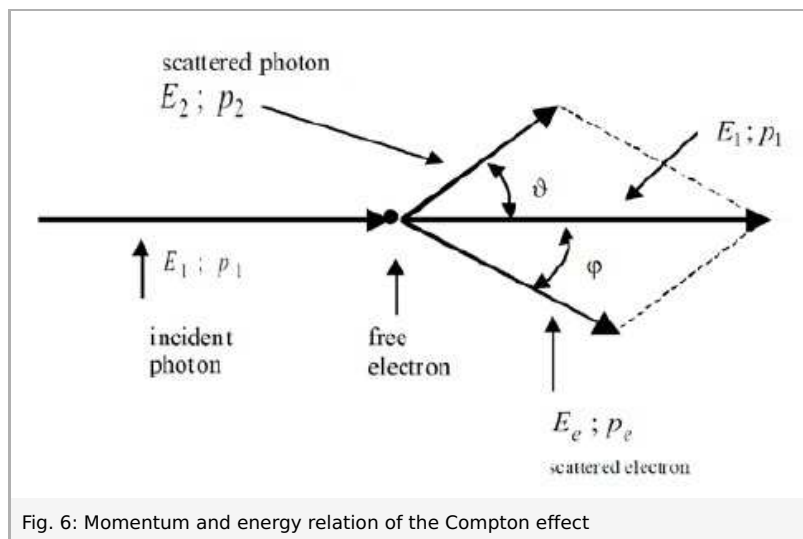
Due to the interaction with a free electron in the solid element, the incident photon loses energy and is deflected from its original direction with the scattering angle ϑ. The previously resting electron absorbs additional kinetic energy and leaves the collision point under the angle φ.

$$E_2 = \frac{E_1}{1+\frac{E_1}{m_0\, c^2}(1-\cos\vartheta)} \tag{1}$$

Photon energy before and after the collision	E_1 resp. E_2
Equivalent	$1\,\text{eV} = 1.6021\cdot10^{-19}\,\text{J}$
Scattering angle	ϑ
Speed of light in vacuum	$c = 2.998\cdot10^8\,\text{ms}^{-1}$
Rest mass of the electron	$m_0 = 9.109\cdot10^{-31}\,\text{kg}$

After the collision, the photon has a lower energy level E_2 and a higher wavelength λ_2.

With $E = h\,\nu$, (1) can be transformed into:

$$\frac{1}{h\,\nu_2} - \frac{1}{h\,\nu_1} = \frac{1}{m_0\,c^2}\left(1 - \cos\vartheta\right) \tag{2}$$

Planck's quantum of action	$h = 6.626\cdot10^{-34}\,\text{Js}$
Photon frequency	ν

With $\lambda = c/\nu$, (2) leads to:

$$\lambda_2 - \lambda_1 = \Delta\lambda = \frac{h}{m_0\,c}\left(1 - \cos\vartheta\right) \tag{3}$$

For the $90°$ scattering, the difference in wavelength, which only consists of the three universal components, provides the so-called Compton wavelength λ_C for electrons.

$$\lambda_C = \frac{h}{m_0\,c} = \frac{6.626\cdot10^{-34}}{9.109\cdot10^{-31}\cdot2.998\cdot10^8}\frac{\text{Js}}{\text{Kg}\cdot\text{ms}^{-1}} = 2.426\,\text{pm}$$

As far as the special cases of the forward and backward scattering by $\vartheta = 0°$ and $\vartheta = 180°$ are concerned, the change in

wavelength is $\Delta\lambda = 2\lambda_C$.

Task 2: Energy determination of the photons of the W-L_α -line that are scattered through an acrylic glass element as a function of the scattering angle.

Figure 7 shows a part of the X-ray spectrum of molybdenum. For the angle-dependent displacement of energy of the scattered radiation, only the high-intensity L_α-line is to be taken into consideration.

Figure 8 shows the tungsten-L_α -Line of various scattering angles ϑ.
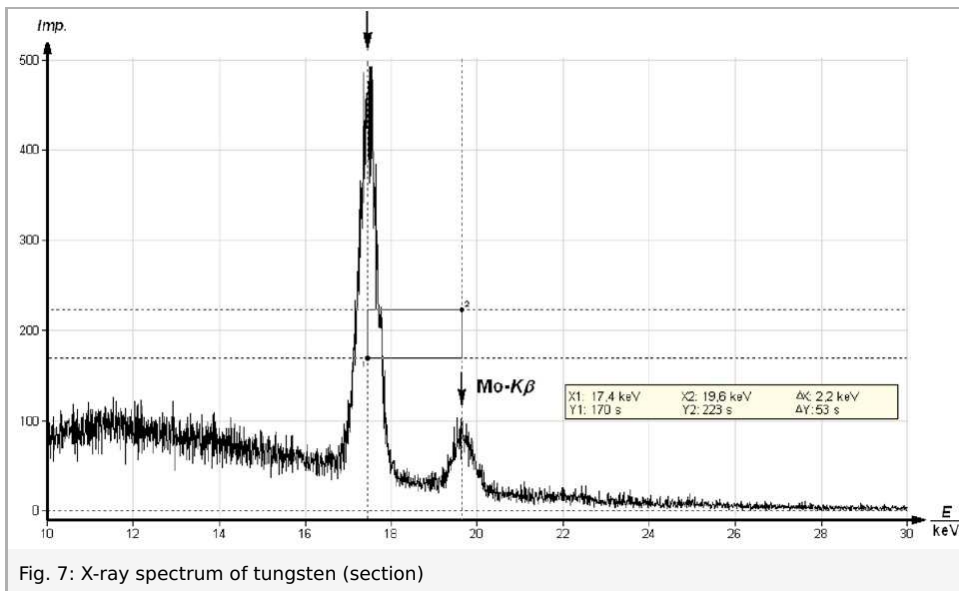

Fig. 7: X-ray spectrum of tungsten (section)

Task 3: Compare the measured energy values of the lines of scatter with the calculated energy values.

Column B of table 1 shows the experimental energy values of the line peaks of the W-L_α-line as a function of the scattering angle (column A).

For comparison, the column C shows the energy values that were calculated with $E_1(\mathrm{Mo} - K_\alpha) = 17.43\,\mathrm{keV}$ based on (1).

Figure 8 shows the content of table 1 in graphical form for clarification.
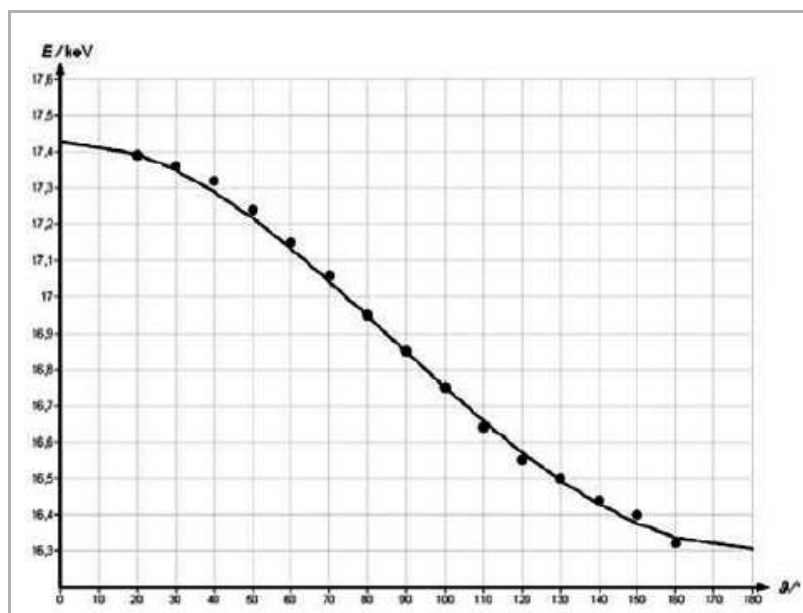

Fig. 8: Energy of the molybdenum K_α -line as a function of the scattering angle.

Extended curve: calculated with $E_1 = 17.43\,\mathrm{keV}$ and equation (1)

- ● = measurement value of column B of table 1

A	B	C
$\vartheta\,/\,°$	E_2 (exp.)/ keV	E_2 (theor.)/ keV
20	17,39	17,394
30	17,36	17,350
40	17,32	17,290
50	17,24	17,218
60	17,15	17,134
70	17,06	17,043
80	16,95	16,947
90	16,85	16,849
100	16,75	16,752
110	16,64	16,659
120	16,55	16,572
130	16,50	16,495
140	16,44	16,429
150	16,40	16,376
160	16,32	16,337

Table 1: Energy E_2 of the scattered photons as a function of the scattering angle ϑ

Task 4: Calculate the Compton wavelength of electrons and a comparison of this value with the corresponding value of the $90°$ scattering.

In order to determine the Compton wavelength λ_C based on the $90°$ scattering, equation (3) is transformed with $\lambda = h \cdot c / E$:

$$\lambda_C = \lambda_2 - \lambda_1 = h \cdot c \left(\frac{1}{E_2} - \frac{1}{E_1} \right) \qquad (4)$$

With $E_2(90°) = 16.64\,\text{keV}$ (see table) and $E_1(0°) = 17.43\,\text{keV}$ and the equivalence $1\,eV = 1.602 \cdot 10^{-19}\,\text{J}$, one obtains the following Compton wavelength based on the experiment:
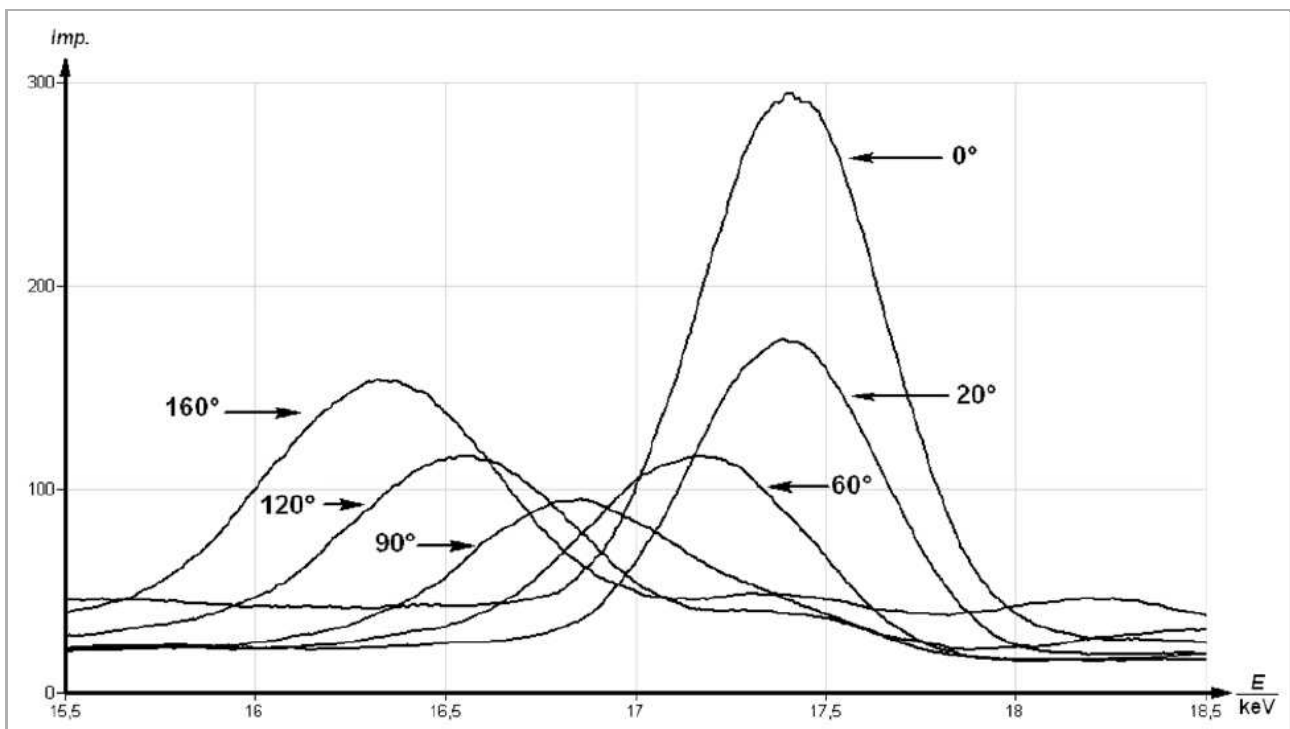
$$\lambda_C = 2.49\,\text{pm}$$

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Fig. 9: Molybdenum-K_α-Line of various scattering angles ϑ

Appendix

Conservation of momentum:

$$p_1 = p_2 + p_3 \rightarrow p_e^2 = p_1^2 + p_2^2 - 2p_1 p_2 \qquad (5)$$

The following applies to the angle ϑ that is formed by the two momentum vectors p_1 and p_2:

$$\cos\vartheta = \frac{p_1 p_2}{\sqrt{\left|p_1^2\right|\left|p_2^2\right|}} \qquad (6)$$

With (6) and the momentum-energy realtions $p_1 = E_1/c$ and $p_2 = E_2/c$ (momentum-energy relation based on a combination of $E = h\nu$; the photon momentum $p = h/\lambda$ (de Broglie) and $c = \lambda\nu$), equation (5) leads to:

$$p_e^2 = \frac{1}{c^2}\left(E_1^2 + E_2^2 - 2E_1 E_2 \cos\vartheta\right) \qquad (7)$$

Conservation of energy:

If one takes the relativistic effects for an electron with the velocity ν into consideration, the following results:

$$E_1 + m_0 c^2 = E_2 + E_e = E_2 + \frac{m_0 c^2}{\sqrt{1 - \nu^2/c^2}} \qquad (8)$$

With $E_e = mc^2$ and $p_e = m\nu$ it follows:

$$\nu^2 = \frac{c^4 p_e^2}{E_e^2} \qquad (9)$$

If one puts (9) into (8), the following results:

$$P_e^2 = \frac{1}{c^2}\left\{E_1^2 + e_2^2 + 2m_0 c^2\left(e_1 - E_2\right) - 2E_1 E_2\right\} \qquad (10)$$

The combination of (7) and (10) leads to the following for E_2:

$$E_2 = \frac{E_1}{1 + \frac{E_1}{m_0 c^2}(1 - \cos\vartheta)}$$

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