Compton scattering of X-rays (Item No.: P2541701)

Curricular Relevance

Area of Expertise:	Education Level:	Topic:	Subtopic:	Experiment:
ILIAS	Physik	Hochschule	Moderne Physik	Röntgenphysik

Difficulty	Preparation Time	Execution Time	Recommended Group Size
Difficult	1 Hour	2 Hours	2 Students

Additional Requirements:

- PC

Experiment Variations:

Keywords:

X-rays, Compton effect, Compton wavelength, rest energy, absorption, transmission, conservation of energy and momentum, Bragg scattering

Overview

Short description

Principle

During this experiment, the Compton wavelength is determined indirectly with the aid of X-rays. For this purpose, X-rays are scattered on an acrylic glass block. The intensity of the scattered radiation is measured with a counter tube. Then, the Compton wavelength is determined based on the transmission behaviour and on a transmission curve that was measured beforehand.

This experiment is included in the upgrade set “XRC 4.0 X-ray characteristics”.

Fig. 1: P2541701

Equipment

Position No.	Material	Order No.	Quantity
1	XR 4.0 expert unit, X-ray unit, 35 kV	09057-99	1
2	XR 4.0 X-ray goniometer	09057-10	1
3	XR 4.0 X-ray Plug-in Cu tube	09057-51	1
4	Geiger-Mueller counter tube, 15 mm (type B)	09005-00	1
5	XR 4.0 X-ray LiF crystal, mounted	09056-05	1
6	XR 4.0 X-ray Compton attachment	09057-04	1
7	XR 4.0 Software measure X-ray	14414-61	1
8	XR 4.0 X-ray Diaphragm tube d = 2 mm	09057-02	1
9	XR 4.0 X-ray Diaphragm tube d = 5 mm	09057-03	1
10	Data cable USB, plug type A/B, 1.8 m	14608-00	1
11	Plate holder	02062-00	1
12	XR 4.0 X-ray optical bench	09057-18	1
13	Slide mount for optical bench expert, h = 30 mm	08286-01	1

Tasks

1. Determine the transmission of an aluminium absorber as a function of the Bragg angle and plot it as a function of the wavelength of the radiation.
2. Measure the intensity of the radiation that is scattered at an angle of a) 60° b) 90° and c) 120° on an acrylic glass block with and without an absorber.
3. Determine the Compton wavelength of the electron based on the transmission curve.

Setup and Procedure

Setup

Connect the goniometer and the Geiger-Müller counter tube to their respective sockets in the experiment chamber (see the red markings in Fig 2). The goniometer block with the analyser crystal should be located in a position in the middle. Fasten the Geiger-Müller counter tube with its holder to the back stop of the guide rails. Do not forget to install the diaphragm in front of the counter tube.
Insert a diaphragm tube with a diameter of 2 mm into the beam outlet of the tube plug-in unit for the collimation of the X-ray beam.

For calibration: Make sure, that the correct crystal is entered in the goniometer parameters. Then, select "Menu", "Goniometer", "Autocalibration". The device now determines the optimal positions of the crystal and the goniometer to each other and then the positions of the peaks.

Fig. 2: Connectors in the experiment chamber

Fig. 3: Connection of the computer

Fig. 4: Part of the user interface of the software

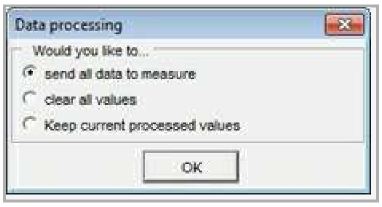
Note

Details concerning the operation of the X-ray unit and goniometer as well as information on how to handle the monocrystals can be found in the respective operating instructions.

Procedure

- Connect the X-ray unit via the USB cable to the USB port of your computer (the correct port of the X-ray unit is marked in Fig. 3).
- Start the “measure” program. A virtual X-ray unit will be displayed on the screen.
- You can control the X-ray unit by clicking the various features on and under the virtual X-ray unit. Alternatively, you can also change the parameters at the real X-ray unit. The program will automatically adopt the settings.
- If you click the experiment chamber (see the red marking in Figure 4), you can change the parameters of the experiments. Select the parameters for task 1 as shown in Figure 5.
- If you click the X-ray tube (see the red marking in Figure 5), you can change the voltage and current of the X-ray tube. Select the parameters as shown in Figure 6.
- Start the measurement by clicking the red circle:

- After the measurement, the following window appears:



- Select the first item and confirm by clicking OK. The measured values will now be transferred directly to the “measure” software.

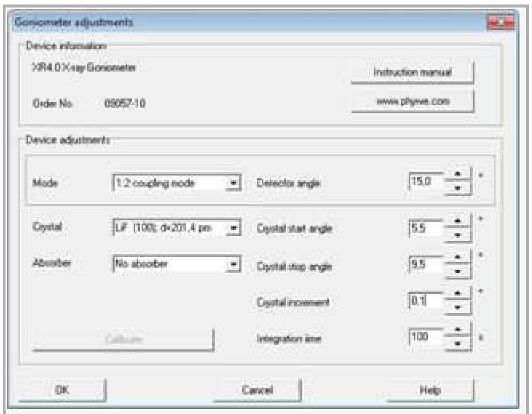


Fig. 5: Settings of the goniometer, task 1

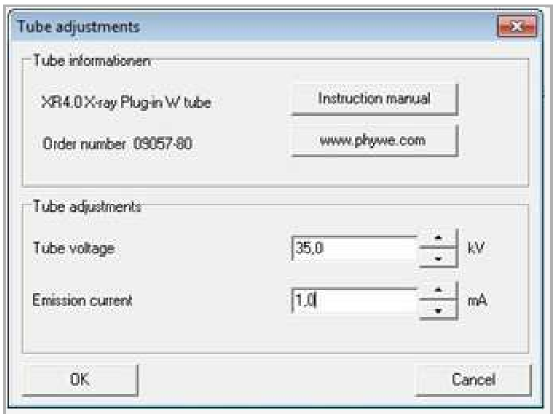


Fig. 6: Voltage and current settings

Overview of the settings of the goniometer and X-ray unit for task 1:

- 2:1 coupling mode
- Gate time 100 s (gate timer); angle step width 0.1°
- Scanning range 5.5° <� ϑ < 9.5°
- Anode voltage U_A = 35 kV; anode current I_A = 1 mA

Theory and Evaluation

Theory

The absorption of a material is determined by three different interaction processes. Their relative contributions depend on the atomic number (nuclear charge number) Z and on the mass number A of the shielding material.

The most important individual processes are:

- Photoelectric effect; attenuation $\approx Z^4/A$
- Compton scattering; attenuation ~ Z/A
- Pair generation; attenuation $\approx Z^2/A$.

As a result, the energy-dependent absorption coefficient of a material μ consists of the absorption coefficient of the pair generation μP_α, of the photoelectric effect μPh, and of the Compton effect μCo. Two additional mechanisms, the nuclear photoelectric effect and the normal elastic scattering, can usually be neglected for the screening effect.
$\mu = \mu P_\alpha + \mu Ph + \mu Co$

For the X-radiation range that we focus on ($E \approx$ 1-100 keV), μP_α can be neglected (see Fig. 9). μPh is also not relevant for this experiment since only electrons, and not photons, are released. As a result, the detector detects nearly exclusively Compton fractions.

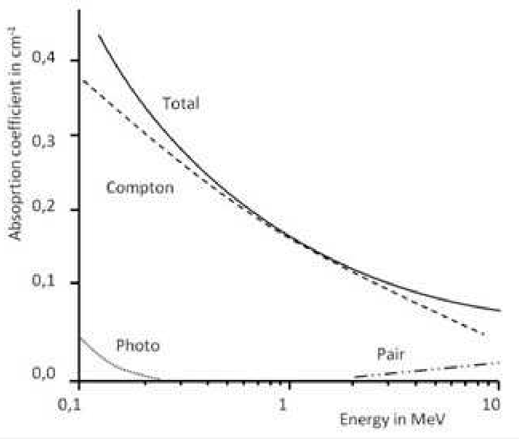


Fig. 9: Absorption coefficient as a function of the energy in the case of aluminium

A schematic representation of the Compton effect is shown in Fig. 10. Due to the interaction with a free electron in the solid material, the incident photon loses energy and is scattered from its original direction under the scattering angle ϑ. The electron that was previously at rest absorbs additional kinetic energy and leaves the collision point under the angle φ.

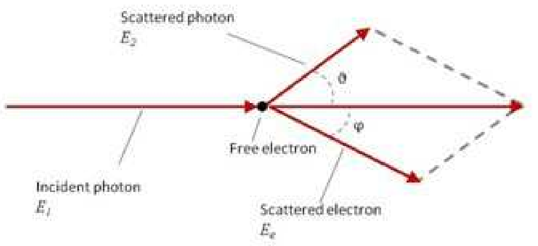


Fig. 10: Momentum and energy relationships in Compton scattering

Based on the principle of conservation of energy and momentum, the energy of the scattered photon is obtained as a function of the scattering angle (see the appendix):

$$E_2 = \frac{E_1}{1+\frac{E_1}{m_0 C^2}(1-cos\vartheta)} \qquad (1)$$

Photon energy before or after the collision	$E_1 or E_2$
Scattering angle	ϑ
Speed of light in vacuum	$c = 2.998 \cdot 10^8 ms^{-1}$
Rest mass of the electron	$m0 = 9.109 \cdot 10^{-31} kg$

After the collision, the photon has a smaller energy E_2 and, therefore, a greater wavelength λ_2 than before the collision. With $E = h\nu$, (1) can be converted into:

$$\frac{1}{hv_2} - \frac{1}{hv_1} = \frac{1}{m_0 C^2}(1 - cos\vartheta \qquad (2)$$

Planck's constant $h = 6.626 \cdot 10^{-34} Js$

Photon frequency ν

With $\lambda = c/\nu$, equation (2) leads to:

$$\lambda_2 - \lambda_1 = \Delta\lambda = \frac{h}{m_0 c}(1 - cos\vartheta) \qquad (3)$$

For 90°-scattering, the wavelength difference, which consists only of the three universal components, leads to the so-called Compton wavelength λ_C for electrons.

$$\lambda_C = \frac{h}{m_0 c} = \frac{6,626 \cdot 10^{-34}}{9,109 \cdot 10^{-31} \cdot 2,998 \cdot 10^8} \frac{Js}{kg \cdot m s^{-1}} = 2,426 pm$$

For the special cases of backward scatter (ϑ = 180°), the change in wavelength is $\Delta\lambda = 2\lambda_C$.

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Evaluation

Task 1: Determine the transmission of an aluminium absorber as a function of the Bragg angle and plot it as a function of the wavelength of the radiation.

Based on the glancing angles ϑ as well as on the Bragg relationship, the associated wavelengths λ are obtained:

$$2dsin\vartheta = n\lambda \qquad (4)$$

with $d = 201.4\,\mathrm{pm}$ = LiF-(200) interplanar spacing and here: $n = 1$.

For a given gate time Δt and the pulse rate n the total number of incidents N is $n \cdot \Delta t$.
For the measured number of incidents N, the relative error of N is given by the ratio:

$$\frac{\Delta N}{N} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}} \qquad (5)$$

At high pulse rates, the dead time τ of the counter tube must also be taken into consideration, since it does not register all of the incident photons. For the GM counter tube that is used in this experiment, it is $90\,\mu\mathrm{s}$.
The true pulse rate $n*$ can be obtained from the measured pulse rate n with the aid of:

$$n^* = \frac{n}{1-\tau n} \qquad (6)$$

Correct the measured count rate in an angle range of $5.5^\circ < \vartheta < 9.5^\circ$ and with the dead time $\tau = 90\,\mu\mathrm{s}$ of the Geiger-Müller counter tube. This can be done using the software measure:

$$T(\lambda) = \frac{n_2^*(\text{with an absorber})}{n_1^*(\text{without an absorber})}$$

It is then plotted as a function of λ (see Fig. 11).

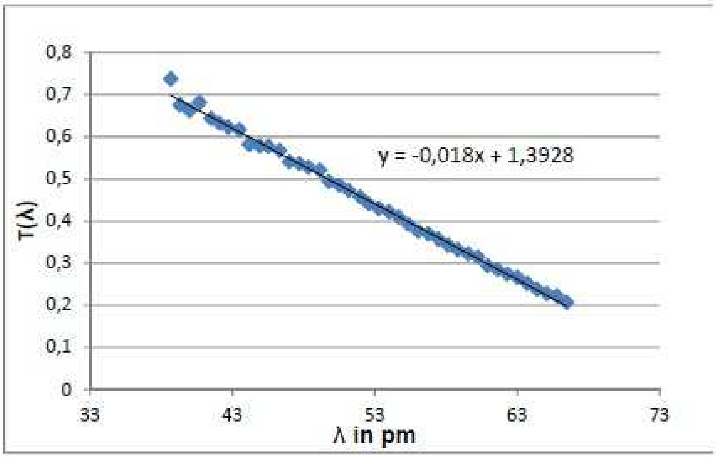

Fig. 11: Transmission curve of aluminium in a narrow wavelength range

Task 2 and 3: Measure the intensity of the radiation that is scattered at an angle of 60°, 90° and 120° on an acrylic glass block with and without an absorber and determine the Compton wavelength of the electron based on the transmission curve.

In this experiment, the aluminium absorber acts as a kind of strong colour filter. It absorbs shorter wavelengths less strongly than longer wavelengths. This means that if it is positioned in front of the scatterer, it has a different effect than in the long-wave scattered radiation behind the scatterer. This enables the determination of the wavelength of the scattered and non-scattered radiation.

By placing the absorber in the ray path between the X-ray tube and the scatterer (position 1, Fig. 8), you can determine the transmission $T_1 = n_4 / n_3$ of the still non-scattered X-radiation. When the absorber is in position 2, you obtain the transmission of the scattered X-radiation.

Since we have determined the dependence of the transmission of aluminium on the wavelength in task 1, we can now directly infer from the transmission to the wavelength of the X-radiation that passes through the absorber. The two different transmission coefficients that are obtained from the 90° scattering ($T_1 > T_2$) then lead to the corresponding wavelengths.

Sample results

Sample results

Fig. 11 shows transmission curve of aluminium in a narrow wavelength range including the equation of the regression line:

$$y = -0.018x + 1.3928$$

The results from Task 2 for $n3*, n4*andn5*$ are listed in table 1

Sample calculation for the 90° scattering:

$$T_1 = \frac{N*_4}{N*_3} = \frac{n*_4 \cdot \Delta\tau}{n*_3 \cdot \Delta\tau} = \frac{83.625 Imp/s}{241.121 Imp/s} = 0.347 \pm 1.27$$

$$T_2 = \frac{N*_5}{N*_3} = \frac{76.017 Imp/s}{241.121 Imp/s} = 0.315 \pm 1.32$$

The deviation of $\pm$ 1.27% and $\pm$ 1.32% is calculated with the aid of equation (2) and the following equation (use the total number of incidents $N* = n* \cdot \Delta\tau$ for the calculation):

$$\Delta T_1 = \sqrt{\left(\frac{1}{\sqrt{\Delta N*_4}}\right)^2 + \left(\frac{1}{\sqrt{\Delta N*_3}}\right)^2}$$

The relative error only takes into account the statistical errors. Systematic errors (see note) are not considered.
Based on the linear equation of the regression line in Fig. 11, the associated wavelengths for the 90° scattering result as 58.93 pm and 61.48 pm. This results in a wavelength difference of $\Delta\lambda = \lambda C = 2.56 pm$, which is very close to the theoretical value of λC = 2.426 pm.

Table 1 sample results

60°	241.121	83.625	76.017	0.347	0.315	58.09	59.87	1.78
90°	178.833	59.315	51.235	0.332	0.286	58.93	61.48	2.56
120°	216.124	70.444	58.81	0.336	0.272	58.71	62.26	3.56

The experiment show that with decreasing scattering angle, the difference in wavelength also de-creases.

Note

Note

There is a systematic error, since

- X-rays are also diffracted at the aluminium sheet so that they might actually enter the counter directly.
- The incident x-radiation is polychromatic
- The geometry of the scattering region is not spherical

The error caused by fluorescence is negligible because the corresponding radiation energy is too low to be registered.