## Coulomb potential and Coulomb field of metal spheres (ttem No.: P2420500)

## Curricular Relevance



## Difficulty <br> 

Difficult

Preparation Time


1 Hour

Execution Time


2 Hours

Recommended Group Size


2 Students

Additional Requirements:

## Keywords:

electric field, field intensity, electric flow, electric charge, Gaussian rule, surface charge density, induction, induction constant, capacitance, gradient, image charge, electrostatic potential, potential difference

## Overview

## Short description

Conducting spheres with different diameters are charged electrically. The static potentials and the accompanying electric field intensities are determined by means of an electric field meter with a potential measuring probe, as a function of position and voltage.


Fig. 1: Experimental set-up used to determine the Coulomb potential.

## Equipment

| Position No. | Material | Order No. | Quantity |
| :--- | :--- | :--- | :--- |
| 1 | Electric field meter | $11500-10$ | 1 |
| 2 | Potential probe | $11501-00$ | 1 |
| 3 | Capacitor plate w.hole d 55 mm | $11500-01$ | 1 |
| 4 | PHYWE High voltage supply unit with digital display DC: $0 \ldots \pm 10 \mathrm{kV}, 2 \mathrm{~mA}$ | $13673-93$ | 1 |
| 5 | Conductor ball, d 20mm | $06236-00$ | 1 |
| 6 | Conductor ball, d 40mm | $06237-00$ | 1 |
| 7 | Conductor ball, d 120mm | $06238-00$ | 1 |
| 8 | High-value resistor, 10 MOhm | $07160-00$ | 1 |
| 9 | Insulating stem | $06021-00$ | 2 |
| 10 | PHYWE power supply DC: $0 \ldots . .12 \mathrm{~V}, 2$ A / AC: $6 \mathrm{~V}, 12 \mathrm{~V}, 5 \mathrm{~A}$ | $13506-93$ | 1 |
| 11 | Multi-range meter, analogue | $07028-01$ | 1 |
| 12 | Barrel base PHYWE | $02006-55$ | 3 |
| 13 | Stand tube | $02060-00$ | 1 |
| 14 | Tripod base PHYWE | $02002-55$ | 1 |
| 15 | Meter scale, demo. I=1000mm | $03001-00$ | 1 |
| 16 | Rubber tubing, i.d. 6 mm | $39282-00$ | 1 |
| 17 | Blow lamp, butan cartridge, X 2000 | $46930-00$ | 1 |
| 18 | Butane cartridge C206, without valve | $47535-01$ | 2 |
| 19 | Connecting cord, $30 \mathrm{kV}, 500 \mathrm{~mm}$ | $07366-00$ | 1 |
| 20 | Connecting cord, $32 \mathrm{~A}, 750 \mathrm{~mm}$, red | $07362-01$ | 3 |
| 21 | Connecting cord, $32 \mathrm{~A}, 750 \mathrm{~mm}$, blue | $07362-04$ | 2 |
| 22 | Connecting cord, $32 \mathrm{~A}, 750 \mathrm{~mm}$, green-yellow | $07362-15$ | 2 |
| 23 | Connecting cord, $32 \mathrm{~A}, 250 \mathrm{~mm}$, green-yellow | $07360-15$ | 2 |

## Tasks

1. For a conducting sphere of diameter $2 R=12 \mathrm{~cm}$, electrostatic potential is determined as a function of voltage at a constant distance from the surface of the sphere.
2. For the conducting spheres of diameters $2 R=12 \mathrm{~cm}$ and $2 R=4 \mathrm{~cm}$, electrostatic potential at constant voltage is determined as a function of the distance from the surface of the sphere.
3. For both conducting spheres, electric field strenght is determined as a function of charging voltage at three different distances from the surface of the sphere.
4. For the conducting sphere of diameter $2 \mathrm{R}=12 \mathrm{~cm}$, electric field strenght is determined as a function of the distance from the surface of the sphere at constant charging voltage.

## Set-up and procedure

## Part 1 : Coulomb potential

The experimental set-up is shown in Fig. 1. The voltage measuring attachment should be mounted on the electric field meter, to which the potential measuring probe is attached. The attachment and the electric field meter must be earthed. The glass rod of the potential measuring probe is connected to the burner with rubber tubing. The burner must be adjusted in such a way that a flame of approx. 5 mm burns over the tip of the probe. This assures ionisation of the air over the tip of the probe, in order to have a volume of conductive air before the probe tip.

The conducting spheres, which are held on insulating supports, are connected to the positive pole of the high voltage power supply by means of a high voltage cable, over the $10 \mathrm{M} \Omega$ safety resistor. The negative pole of the power supply is earthed.

To begin with, the electrostatic potential of a charged sphere is determined as a function of voltage. This is done by applying voltages in steps of 1 kV up to maximum of 5 kV to the sphere with diameter $2 \mathrm{R}=12 \mathrm{~cm}$. During this procedure, the measuring probe should be situated about 25 cm from the surface of the sphere.
To determine the potential as a function of the distance from the centre of the sphere, a 1000 V steady voltage is applied to the sphere with $2 R=12 \mathrm{~cm}$. This voltage can be measured directly with the electric field meter, the probe tip of which touches the surface of the sphere. At the beginning, the height adjusted tip of the measuring probe is situated 1 cm from the surface of the sphere. The potential is then measured for steps of 1 cm . Measurement is repeated for the conducting sphere of $2 R=4 \mathrm{~cm}$.

## Part 2 : Coulomb field

The experimental set-up is modified according to Fig. 2 . The conducting sphere ( $2 \mathrm{R}=2 \mathrm{~cm}$ ), held on an insulated support, is connected to the positive pole of the high voltage power supply, as described above, in order to charge the test spheres.


Fig. 2: Experimental set-up to determine the Coulomb field.
The height of the electric field meter with attached capacitor plate is adjusted by means of the clamp stand, so that its axis lies in the equator plane of the test sphere. The stem of the electric field meter is earthed again.
To determine field strenght as a function of the charging voltage, the surface of the conducting sphere of $2 \mathrm{R}=12 \mathrm{~cm}$ is placed successively at distances $r_{1}=25 \mathrm{~cm}, r_{2}=50 \mathrm{~cm}$ and $r_{3}=75 \mathrm{~cm}$ of the electric field meter capacitor plate.

The test sphere is charged to a maximum voltage of 10 kV in steps of 1 kV . After every charging procedure, the high voltage is to be set back to zero volts; after every measurement, the test sphere is to be discharged by briefly touching it with the earth lead. The measurement series is repeated with the $2 R=4 \mathrm{~cm}$ sphere for $r=25 \mathrm{~cm}$.

To determine field strenght as a function of distance $r$ from the centre of the sphere, conducting sphere $2 \mathrm{R}=12 \mathrm{~cm}$ is charged to 10 kV and the electric field meter is set up in steps of 5 cm from the surface of the sphere.

After charging, high voltage is to be reset to zero, in order to avoid disturbing influences.

## Theory and evaluation

At a distance rof its surface, the electric potential of a charged conducting sphere is:
$\varphi(r)=\frac{1}{4 \pi \cdot \varepsilon_{0}} \frac{Q}{r}$
( $Q=$ electric charge; $\varepsilon_{0}=$ induction constant)
If the capacitance of the sphere is Cand its radius $R$, charge $Q$ at voltage $U$ is given by:
$Q=C U=4 \pi \varepsilon_{0} \cdot R \cdot U(2)$
Introducing (2) into (1) yields the following relation:
$\varphi(r)=\frac{R}{r} \cdot U$ (3)
According to (3), Fig. 3 shows the linearity of potential $\varphi=\varphi(\mathrm{U})$ for $r=$ const. $=18 \mathrm{~cm}$, as measured on the conducting sphere with $2 R=12 \mathrm{~cm}$.


Fig. 3: Potential as a function of voltage $U$.

Calculating the logarithm of (3), one obtains
$\log \varphi=-\log r+\log R+\log U=-\log r+k$ (4)
if $U$ and $R$ are constant. (Straight line with slope $m=-1$ ).
In Fig. $4, \varphi$ is plotted against ron a double logarithmic scale, rbeing measured from the centre of the sphere. Fig. 4a shows the result of measurement on the conducting sphere with diameter $2 R=12 \mathrm{~cm}$, Fig. 4b displays the corresponding result for the sphere with diameter $2 R=4 \mathrm{~cm}$. The voltage applied to the spheres was 1000 V each time. In both cases, the slope of the straight line was $m \cong-1$, in agreement with (4); this is thus the experimental proof of the $1 / r$-dependence of the electric potential. For points situated near the surface of the spheres, non-linear measurement values were found, due to the influence of the flame on the tip of the probe.



Fig. 4: Potential as a Function of distance $r$ represented in double logarithmic scale. Fig. 3a: Sphere with $2 r=12 \mathrm{~cm}$; Fig. 3b: Sphere with $2 r=$ 4 cm .

If the potential field $\varphi(r)$ is known, the electric Coulomb field Efor a charge Qis obtained from the negative gradient of the potential:
$E=-\operatorname{grad} \varphi=-\mathrm{d} \varphi / \mathrm{d} r=\frac{1}{4 \pi \varepsilon_{0}} \frac{Q}{r^{2}}(5)$
To determine the field strenght, a capacitor plate is placed before the electric field meter in order to obtain a spatially undistorted field distribution. This is shown schematically in Fig. 5. An image charge is obtained inductively by means of the plate, so that the plate is situated centrally between the real and the virtual charge. The charge Qin (5) must thus be multiplied by 2 .


Replacing the value of $Q$ in (5) by (2) and considering the doubling of the charge, the following holds true:
$E=\frac{2 R}{r^{2}} \cdot U(6)$
The measured field strength Eis plotted as a function of voltage Uin Fig. 6.


Fig. 6: Field strenght as a function of voltage. Graphs 1-3: sphere with $2 R=12 \mathrm{~cm} ; r_{1}=25 \mathrm{~cm}, r_{2}=50 \mathrm{~cm}, r_{3}=75 \mathrm{~cm}$; graph 4: sphere with $2 R=4 \mathrm{~cm} ; r_{1}=25 \mathrm{~cm}$

A comparison of the slope $\Delta E / \Delta U$ of the straight lines with the corresponding quotients $2 R / r^{2}$ yields the following pairs of values which agree satisfactorily:
graph 1: $\Delta E / \Delta U=1.44 \mathrm{~m}^{-1}, 2 R / r^{2}=1.25 \mathrm{~m}^{-1}$
graph 2: $\Delta E / \Delta U=0.44 \mathrm{~m}^{-1}, 2 R / r^{2}=0.38 \mathrm{~m}^{-1}$
graph 3: $\Delta E / \Delta U=0.18 \mathrm{~m}^{-1}, 2 R / r^{2}=0.18 \mathrm{~m}^{-1}$
graph 4: $\Delta E / \Delta U=0.58 \mathrm{~m}^{-1}, 2 R / r^{2}=0.55 \mathrm{~m}^{-1}$

In Fig. 7, field strenght $E$, measured on the sphere with radius $2 R=12 \mathrm{~cm}$ charged at 10 kV , is plotted on a double logarithmic scale as a function of distance $r$.


Fig. 7: Field strenght Eas a function of distance rin double logarithmic scale. Sphere with $2 R=12 \mathrm{~cm}$.
The slope of the straight line is $m=-2.06$. This means that the field strenght is inversely proportional to the square of the distance.

