TESS expert **PHYWE**

Coulomb's law / Image charge (Item No.: P2420405)

Curricular Relevance
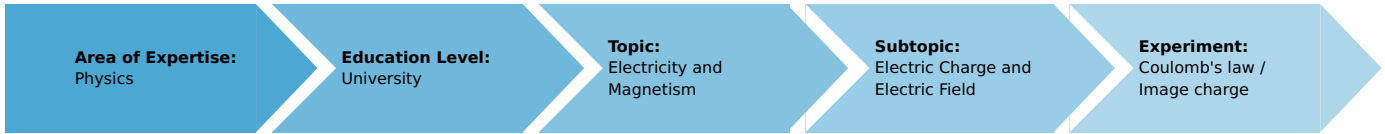
| Area of Expertise: Physics | Education Level: University | Topic: Electricity and Magnetism | Subtopic: Electric Charge and Electric Field | Experiment: Coulomb's law / Image charge |

Difficulty	**Preparation Time**	**Execution Time**	**Recommended Group Size**
Intermediate	1 Hour	2 Hours	2 Students

Additional Requirements:	**Experiment Variations:**
	• P2420401 Coulomb's law with the DC measuring amplifier

Keywords:

Electric field, Electric field strength, Electric flux, Electrostatic induction, Electric constant, Surface charge density, Dielectric displacement, Electrostatic potential

Overview

Short description

Principle

A small electrically charged ball is positioned at a certain distance in front of a metal plate lying at earth potential. The surface charge on the plate due to electrostatic induction together with the charged ball forms an electric field analogous to that which exists between two oppositely charged point charges.

The electrostatic force acting on the ball can be measured with a sensitive torsion dynamometer.
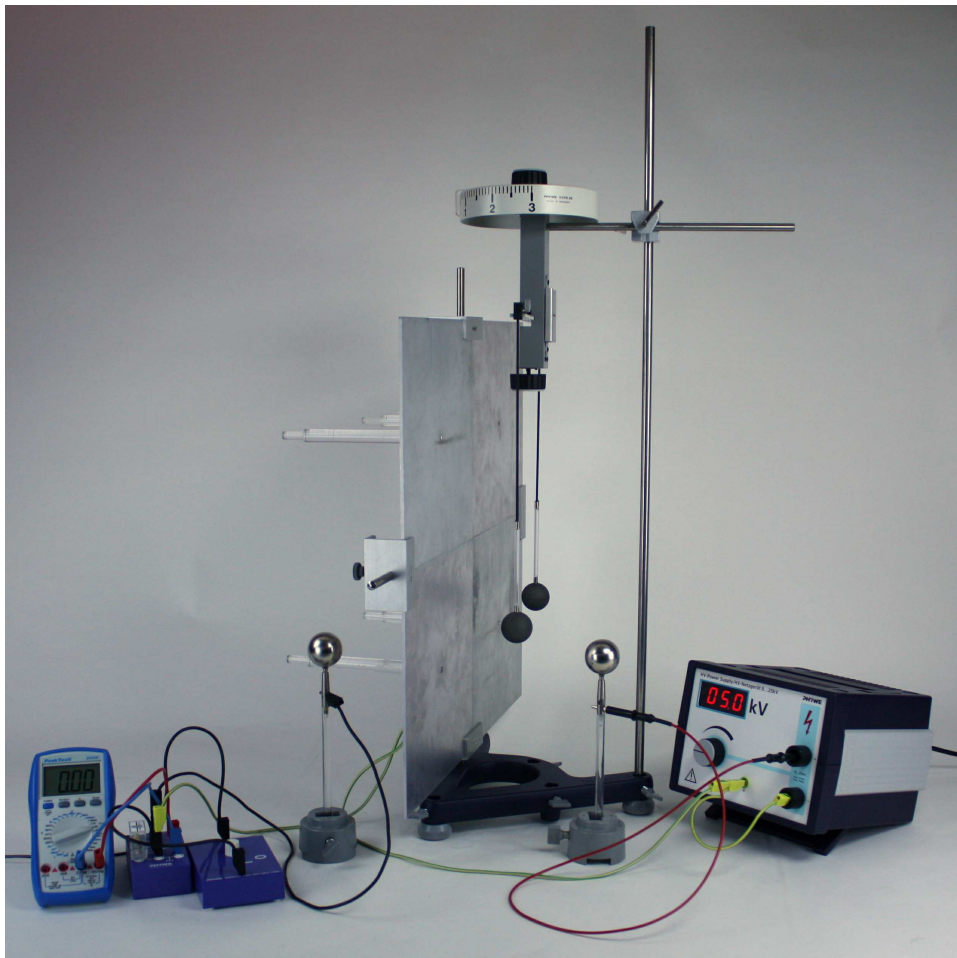
Fig. 1: Experimental set-up for measurement of the electrostatic force of attraction acting on the ball.

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Equipment

Position No.	Material	Order No.	Quantity
1	PHYWE Power supply, high voltage DC: 0... ± 25 kV, 0,5 mA	13671-93	1
2	Support base DEMO	02007-55	1
3	Support rod, stainless steel, 1000 mm	02034-00	1
4	Right angle clamp expert	02054-00	1
5	Torsion dynamometer, 0.01 N	02416-00	1
6	Conductor spheres, w. suspension	02416-01	2
7	Plate capacitor, 283x283 mm	06233-02	4
8	Holder for U-magnet	06509-00	4
9	Barrel base expert	02004-55	2
10	Insulating stem	06021-00	2
11	Conductor ball, d 40mm	06237-00	2
12	Connecting cord, 30 kV, 1000 mm	07367-00	1
13	Connecting cord, 32 A, 1000 mm, black	07363-05	1
14	Electrometer Amplifier	13621-00	1
15	Digital multimeter 2005	07129-00	1
16	On/off switch	06034-01	1
17	Short-circuit plug, white	06027-06	1
18	Capacitor 100 nF/250V, G1	39105-18	1
19	Connecting cord, 32 A, 1000 mm, green-yellow	07363-15	2
20	Connecting cord, 32 A, 250mm, green-yellow	07360-15	1
21	Connecting cord, 32 A, 250 mm, black	07360-05	2
22	Connecting cord, 32 A, 500 mm, red	07361-01	1
23	Connecting cord, 32 A, 500 mm, blue	07361-04	1
24	Measuring tape, l = 2 m	09936-00	1

Tasks

Duration: approx. 30 minutes for set-up, 60 minutes for performance and 30 minutes for evaluation

Tasks

1. Establishment of the relation between the active force and the charge on the ball.
2. Establishment of the relation between force and distance, ball to metal plate.
3. Determination of the electric constant.

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Set-up and procedure

Set-up

Set up the equipment as seen in Fig. 1 and 2. The manual includes two conducting spheres, that hang from the dynamometer and two conductor balls that are placed on insulating stems. Both of the conductor spheres are attached to the torsion dynamometer. The one in the center in front of the plate is used for the measurement, the other one acts only as a counterbalance.
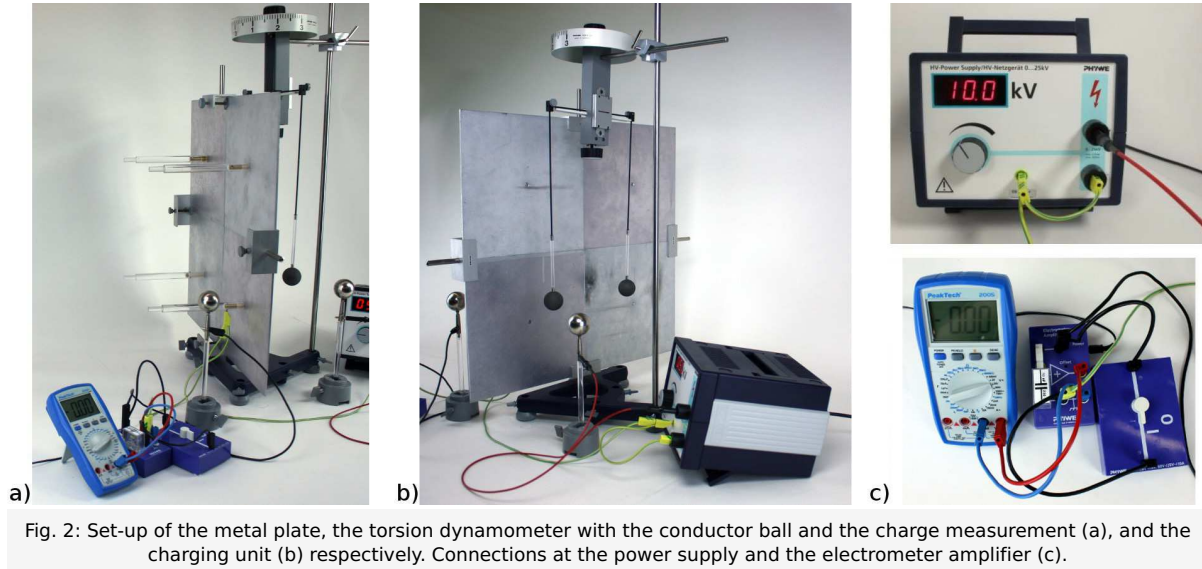

Fig. 2: Set-up of the metal plate, the torsion dynamometer with the conductor ball and the charge measurement (a), and the charging unit (b) respectively. Connections at the power supply and the electrometer amplifier (c).

One of the conductor balls on the insulating stem is used for charging the conductor sphere. It is connected to the "+" port of the high voltage power supply using the high voltage connecting cord. It is used to charge the conductor sphere that is hanging from the dynamometer. To do so, touch the conductor sphere with the charged conductor ball. As soon as the conducting sphere is charged, the conductor ball is placed somewhere on the table.
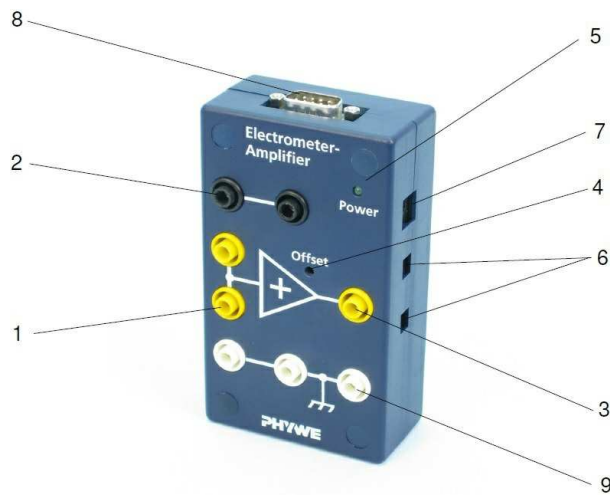

Fig. 3: Connections of the electrometer amplifier

Connect the connector (7) of the electrometer amplifier (Fig. 3) to the power supply. Put the bridge ("short-circuit plug") between (1) and (2) and the 10 nF capacitor between (1) and (9). Keep the electrometer amplifier's input earthed by connecting (9) to the ground of the HV power supply.

Keep any high voltage away from the electrometer amplififer, its input is for up to 10 V, only!

The conductor ball on the insulating stem for measuring the charge is connected to the auxiliary input (2) of the electrometer measuring amplifier. It is used to measure the charge at the conducting sphere after the experiment. To do so, touch the charged conductor sphere with this conductor ball for measurement.

Connect the on/off switch to (2) and (9) of the electrometer amplifier. By closing the switch, the capacitor will be discharged after each measurement. Keep the switch always closed and open it only when collecting and measuring the charge of the suspended sphere.

Select the measurement range of the multimeter to 20 V DC. Connect the blue cable to COM port of the multimeter and to the reference potential (9) of the electrometer amplifier. Connect the red cable to the V port of the multimeter and the amplifier

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output (3) of the electrometer amplifier.

While the acting force is 0 (Fig. 4, left), use the turning knob at the bottom of the torsion dynamometer (Fig. 4, mid) to set its arms to their initial position (Fig. 4, right). The arms of the torsion dynamometer should be mounted in a parallel way to the plate.
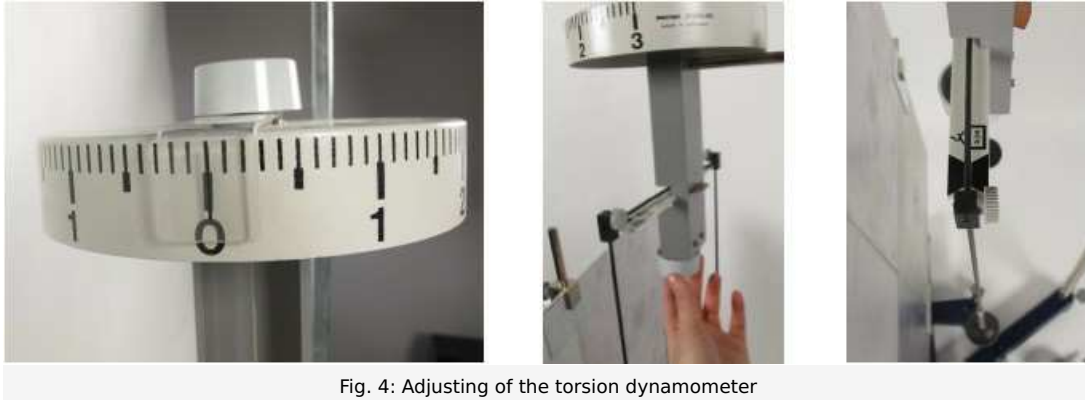

Fig. 4: Adjusting of the torsion dynamometer

Procedure

Set the distance a between the center of the conducting sphere and the plates to 4 cm. Set the high voltage U to 5 kV. After charging the conducting sphere, it moves towards to the plate.

Now bring the arm of the dynamometer back to its initial position using the upper turning knob to apply a counterforce. After restoring the initial position, measure immediately the charge Q of the conducting sphere using the electrometer amplifier (use $Q = C \cdot U$ with $C = 10 \, \text{nF} = 10^{-8} \, \text{F}$) and read the applied force F. Note all values.

After noting the value of the charge, close the on/off switch to discharge the measuring capacitor and to set the reading of the multimeter back to 0.

Measuring the charge means also discharging of the conducting sphere. For that reason it moves away from the plate. Keep this position and increase the voltage to 10 kV and repeat the steps mentioned above.

Perform the same procedure also for 15, 20 and 25 kV.

After completing, set the applied force back to 0 and check if the initial position for the discharged conducting sphere is still the same.

Increase the distance to 8 cm in steps of 1 cm and measure again for each step. At distances of 7 cm and 8 cm, you may find no deflection at low voltages.
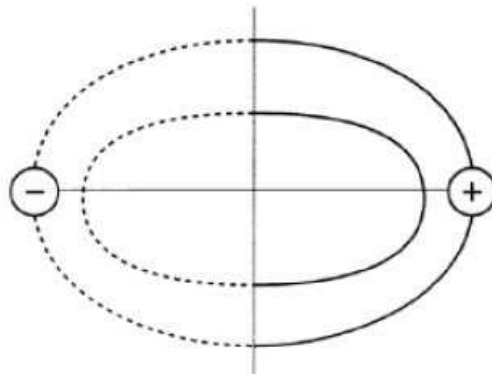
Theory and evaluation

Theory

Fig. 5: Principle for Coulomb's law and image charge.

In accordance with Fig. 6 the electrostatic potential φ in the vicinity of two point charges of opposite polarity in the point P defined by $\vec{r}$ is
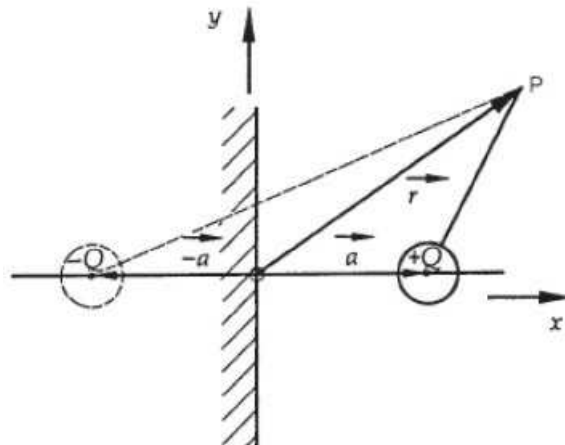

Fig. 6: Geometrical relationship in the plate/charge and image charge/charge system.

$$\varphi(\vec{r}) = \frac{Q}{4\pi\varepsilon_0|\vec{r} - \vec{a}|} - \frac{Q}{4\pi\varepsilon_0|\vec{r} + \vec{a}|} = \frac{Q}{4\pi\varepsilon_0\sqrt{(x-a)^2 + y^2}} - \frac{Q}{4\pi\varepsilon_0\sqrt{(x+a)^2 + y^2}}$$

where Q represents the amount of charge and ε_0 the electric constant. To prove this spatial potential distribution in the plate/ball system, it is advisable to relate the electrostatic potential to a certain locus (e.g. $1/2\,\vec{a}$). We obtain

$$\varphi(\vec{r}) = \frac{3}{4}\varphi\left\{\frac{1}{2}\vec{a}\left(\frac{a}{\sqrt{(x-a)^2 + y^2}} - \frac{a}{(x+a)^2 + y^2}\right)\right\}$$

In an example of measurement there is a potential of 1000 V with respect to earth potential on the conductor ball. One obtains for the reference point in this case:

$$\varphi(\tfrac{1}{2}\vec{a}) = 175 \text{ V}$$

See experiment P2420100 "Electric fields and potentials in the plate capacitor".

From

$$\vec{E}(\vec{r}) = -\text{grad}\frac{-Q}{4\pi\varepsilon_0|\vec{r} - \vec{a}|}$$

The electrostatic field produced by the image charge becomes

$$\vec{E}(\vec{r}) = -\frac{-Q}{4\pi\varepsilon_0|\vec{r} + \vec{a}|^3}(\vec{r} - \vec{a})$$

Hence the electrostatic force acting on the charge at the locus

$$\vec{F} = Q(\vec{E}(\vec{a})) = -F\frac{\vec{a}}{a}$$

with

$$F = \frac{Q}{16\pi\varepsilon_0 a^2}$$

Evaluation

Task 1:

The pairs of values of force and charge found for different distances a between the conductor ball and the condenser plate in a measurement example are listed in Table 1.

Table 1: Measurement of charge Q and force F for different distances.
*For the distance $a = 8\,\text{cm}$ the forces for the voltages 5 kV and 10 kV have been too small for measurement.

	Distance: 4 cm		Distance: 5 cm		Distance: 6 cm		Distance: 7 cm		Distance: 8 cm	
U in kV	Q in nAs	F in mN	Q in nAs	F in mN	Q in nAs	F in mN	Q in nAs	F in mN	Q in nAs	F in mN
5	5.1	0.03	4.4	0.04	4.5	0.02	4.5	0.02	–	–
10	10.2	0.18	9.2	0.09	8.2	0.08	8.3	0.08	–	–
15	15.5	0.39	13.8	0.19	13.2	0.12	12.4	0.11	12.4	0.07
20	20.1	0.65	18.0	0.32	17.1	0.19	16.8	0.16	16.0	0.11
25	23.8	0.94	21.8	0.46	23.0	0.36	21.0	0.24	20.9	0.16

The force F is proportional to the square of the charge:

$$F = A_a \times Q^2 + b$$

The intercept b is needed for a flexible fit of the data and can be neglected. In an ideal or theoretical case, $b = 0$.

Task 2:

The relationship between electrostatic force F and the square of the charge Q is plotted and the proportionality factors A_a between F and Q^2 for each distance a is determined form the slope of the straight line. It is a function of the distance a between condenser plate and ball.
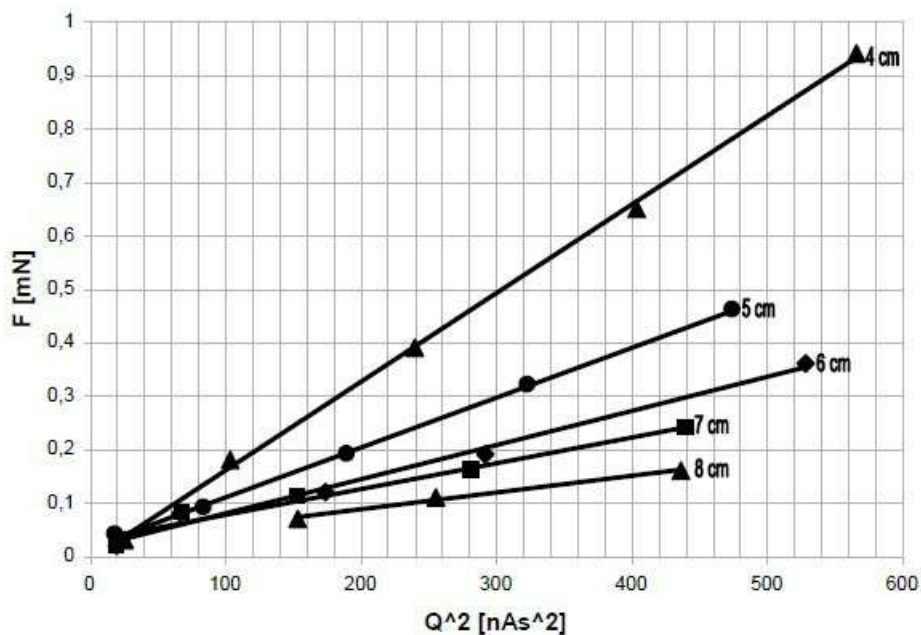

Fig. 6: Charge Q^2 and force F for different distances a

Pay attention to the right dimensions like changing the distance from cm to m. Especially, you should calculate A_a carefully,

since its unit is $10^{-12}\,N/(C^2) = 10^{-12}\,V/(m\,As)$.

Table 2: Distances a and the related proportionality factors A_a:

Distance a in m	$1/a^2$ in $1/m^2$	Proportionality factor A_a in 10^{12} V (m As)
0.08	156	0.31
0.07	204	0.48
0.06	278	0.64
0.05	400	0.93
0.04	625	1.66

Task 3:

Finally from these proportionality factors the electric constant can be determined:

The inverse distance $1/a^2$ is plotted and the slope S of the graph is determined (Fig 7).
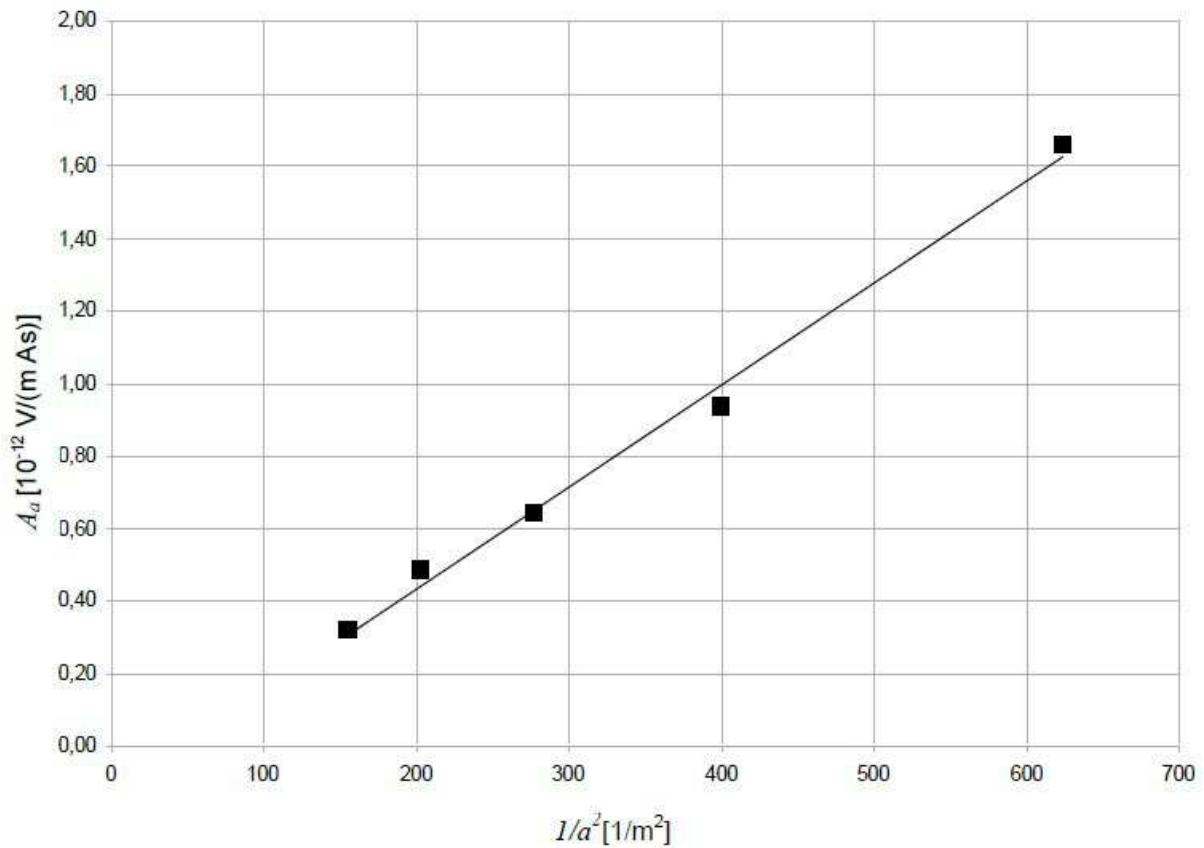

Fig. 7: Determination of the slope S

Slope $S = 0.0028 \cdot 10^{12}\,\mathrm{Vm/As}$

According to: $\varepsilon_0 = \dfrac{Q^2}{16\pi a^2 F}$ it follows: $\varepsilon_0 = \dfrac{1}{16\pi S} = 7.1 \cdot 10^{-12}\,\mathrm{As/Vm}$

The literature value is $\varepsilon_0 = 8.8542 \cdot 10^{-12}\,\mathrm{As/Vm}$.

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