TESS
expert | PHYWE

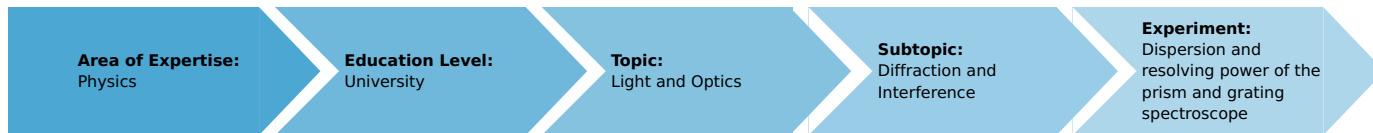# Dispersion and resolving power of the prism and grating spectroscope (Item No.: P2210300)

Curricular Relevance

| Area of Expertise: Physics | Education Level: University | Topic: Light and Optics | Subtopic: Diffraction and Interference | Experiment: Dispersion and resolving power of the prism and grating spectroscope |

Difficulty	**Preparation Time**	**Execution Time**	**Recommended Group Size**
Difficult	1 Hour	3 Hours	2 Students

Additional Requirements:	**Experiment Variations:**

Keywords:

Maxwell relationship, dispersion, polarizability, refractive index, prism, Rowland grating, spectrometer-goniometer

Overview

Short Description

Principle:

The refractive indicies of liquids, crown glass and flint glass are determined as a function of the wavelength by refraction of light through the prism at minimum deviation. The resolving power of the glass prisms is determined from the dispersion curve.

Fig. 1: Experimental set-up for determining dispersion in liquids.

PHYWE excellence in science

Robert-Bosch-Breite 10
D - 37079 Göttingen

Tel: +49 551 604 - 0
Fax: +49 551 604 - 107

info@phywe.de
www.phywe.com

Equipment

Position No.	Material	Order No.	Quantity
1	PHYWE Spectrometer-Goniometer with vernier SG1 PRO	35635-03	1
2	Lamp holder pico 9 for spectral lamps	08119-00	1
3	Spectral lamp Hg 100, pico 9 base	08120-14	1
4	Power supply for spectral lamps	13662-97	1
5	Diffraction grating, 4 lines/mm	08532-00	1
6	Diffraction grating, 8 lines/mm	08534-00	1
7	Grating, 80 lines/mm	09827-00	1
8	Diffraction grating, 600 lines/mm	08546-00	1
9	Magnifier with handle, 4x, d=50 mm	87004-03	1
10	Tripod base PHYWE	02002-55	1
11	Slit, adjustable	08049-00	1
12	Barrel base PHYWE	02006-55	1
13	Right angle clamp expert	02054-00	1
14	Support rod, stainless steel, l = 250 mm, d = 10 mm	02031-00	1

Task

1.   To adjust the spectrometer-goniometer.
2.   To determine the refractive index of various liquids in a hollow prism.
3.   To determine the refractive index of various glass prism.
4.   To determine the wavelengths of the mercury spectral lines.
5.   To demonstrate the relationship between refractive index and wavelength (dispersion curve).
6.   To calculate the resolving power of the glass prisms from the slope of the dispersion curves.
7.   Determination of the grating constant of a Rowland gratin based on the diffraction angle (up to the third oder) of the high intensity spectral lines of mercury.
8.   Determination of the angular dispersion of a grating.
9.   Determination of the resolving power required to separate the different Hg-Lines. Comparison with theory.

PHYWE excellence in science

Robert-Bosch-Breite 10
D - 37079 Göttingen

Tel: +49 551 604 - 0
Fax: +49 551 604 - 107

info@phywe.de
www.phywe.com

Set-up and procedure

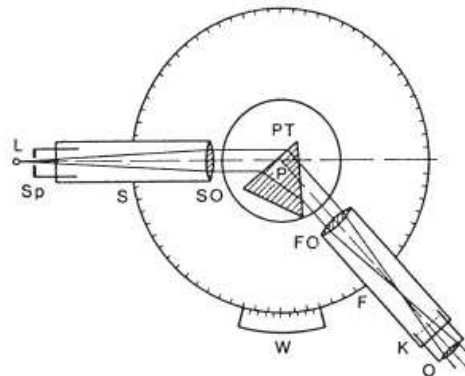## Experiment with the glas prism

Fig. 2: Set-up and path of rays in the spectrometer. (L = light source, Sp = slit in drawtube, S = collimator, SO = collimator lens, PT = prism table with adjust-ing screws, P = prism, FO = telescope lens, F = telescope, O = eyepiece, K = cross-wires, W = graduated circle with vernier)

The experiment is set up as shown in Fig. 1.
The spectrometer-goniometer and the grating should be adjusted in accordance with the operating instructions. When the adjustment is correct, a parallel beam of light will pass through the prism (Fig. 2).
The aperture, or slit, is projected into the plane of the crosswires with the telescope set to infinity and observed with the eyepiece which is used as a magnifier.
The prism is then set to the minimum deviation and the angular position 1 of the telescope is read off on the vernier for each spectral line. The prism is then turned so that the light falls on the adjacent surface and is deviated to the opposite side. The angle 2 is now read off for each spectral line, at minimum deviation.
A ruled grating which is secured in a holder perpendicular to the collimator axis, and takes the place of the prism, is used to determine the wavelengths of the mercury spectral lines. The angles of first-order diffracted lines are measured to the right and left of the undeviated image of the slit.

The spectral lamp reaches its maximum lumiosity after approx. 5 minutes' warm-up time. When setting up the lamp, ensure that air can circulate unimpeded through the ventilation slots on the lamp housing.

PHYWE excellence in science

Robert-Bosch-Breite 10
D - 37079 Göttingen

Tel: +49 551 604 - 0
Fax: +49 551 604 - 107

info@phywe.de
www.phywe.com

Experiment with the grating

Fig. 3: Experimental set-up with vernier used auxiliary slit to determine the resolving power.

The experimental set-up is shown in Fig. 3. To start with, the telescope is adjusted to infinite distance. Then both tubes are adjusted horizontally with the adjusting screws and finally they are adjusted so that the directions of their axes coincide. The Hg-lamp is placed directly before the slit and must illuminate it completely. A sharp image of the slit is formed in the plane of the eyepiece scale and is observed using the eyepiece lens as a magnifying lens. The slit should be selected as narrow as possible. To start with, the grating constant of the high resolution Rowland grating is determined. For this, the grating is set perpendicular to the collimator axis and the grating table is fixed. The diffraction angles of the 6 high intensity Hg spectral lines are determined for the first and second order. Furthermore, recognizable third order lines should also be evaluated. The angle 2 of a spectral line of the same order of diffraction is measured to the right and to the left of the zero order. Two measurement readings are taken for every angle (two verniers).

Usually, the eyepiece scale is difficult to see, due to reduced brightness for higher orders diffraction. In these cases, better visibility may be obtained by lighting the grating askew from the direction of the telescope with a torch light.

The number of illuminated grating slits is reduced to determine the resolving power of the grating. For this purpose, a slide caliper is placed as an auxiliary slit in front of the collimator lens in such a way, that no light reaches the grating when the caliper is closed (cf. Fig. 3).The auxiliary slit is then opened so that for example the yellow and green lines of Hg can be observed as clearly separate lines. The width x of the auxiliary slit is then reduced until the two lines merely appear separated. The average width of the auxiliary slit is determined over several experimental runs. Gratings with up to 50 lines/mm are used to determine the resolution required for the yellow-green lines. The Rowland grating is used to separate the pair of yellow Hg lines. (Cf. Table 4 for grating type and useful order of diffraction)
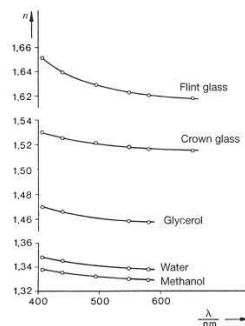

Fig. 4: Dispersion curves of various substances.

Theory and evaluation

Dispersion and resolving power of the prism

The refractive index of a medium is linked to the relative permittivity ε_r by the Maxwell relationship

$$n = \sqrt{\varepsilon_\mathrm{r}\ \mu_\mathrm{r}} \qquad (1)$$

For most substances the permeability is:
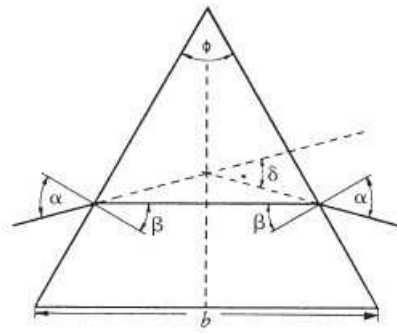
$$\mu_\mathrm{r} = 1.$$

Fig. 5: Refraction by the prism when the path of a ray is symmetrical.

According to Clausius and Mossotti, the following relationship exists between the relative permittivity and the molecular polarizability α of a medium:

$$\alpha = \frac{3\,\varepsilon_0}{N} \cdot \frac{\varepsilon - 1}{\varepsilon + 2} \qquad (2)$$

where N is the concentration of the polarizable molecules and ε_0 is the electric field constant.

The polarizability depends on the frequency $\omega = 2\pi\nu$ of the incident light. The following is approximately true, beyond the natural frequency $\omega_0 = 2\pi\nu_0$ of an atom or molecule:

$$\alpha = \frac{e^2}{m} \cdot \frac{1}{\omega_0^2 - \omega^2} \qquad (3)$$

where e is the elementary charge and m is the mass of an electron.
When (1) and (3) are substituted in (2) we obtain

$$\frac{n^2 - 1}{n^2 + 2} \cdot \frac{e^2 \cdot N}{3\,\varepsilon_0\;m} \cdot \frac{1}{\omega_0^2 - \omega^2} \qquad (4)$$
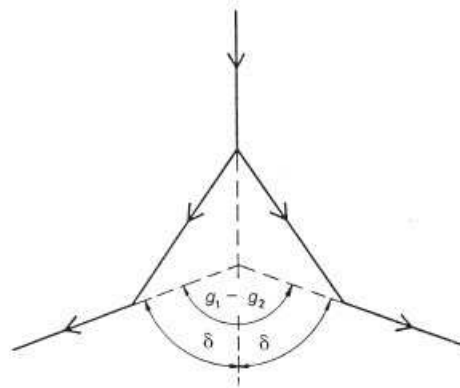

Fig. 6: Measurement of the angle of minimum deviation.

Although equation (4) only takes one natural frequency into account, this formula adequately describes the decrease in the refractive index as the wavelength increases, outside the range of natural frequencies.

The wavelength of the spectral lines are determined with a diffraction grating which is placed in the path of the rays, instead of the prism. For a wavelength λ, the grating constant G and angle ϕ at which the first order diffraction pattern appears, the following applies:

$$\lambda = G \cdot sin\,\phi.$$

λ is determined from the average of several measurements:

$$\begin{aligned}
\lambda_{red} &= 627.3\ nm \\
\lambda_{yellow} &= 579.8\ nm \\
\lambda_{green} &= 547.7\ nm \\
\lambda_{turquoise} &= 493.9\ nm \\
\lambda_{blue} &= 438.5\ nm \\
\lambda_{violet} &= 405.1\ nm.
\end{aligned}$$

PHYWE excellence in science

Robert-Bosch-Breite 10
D - 37079 Göttingen

Tel: +49 551 604 - 0
Fax: +49 551 604 - 107

info@phywe.de
www.phywe.com

If a ray of light passes symmetrically through a prism (Fig. 5), minimum deviation δ occurs.
If α is the angle of incidence, β the angle of reflection, and θ the angle of the prism then

$$sin\ \alpha = n \cdot sin\ \beta$$

$\beta = \frac{\theta}{2}$ and $\delta = 2\alpha - \theta$ \qquad (5)

From these we obtain

$$n = \frac{sin\ \frac{\theta+\delta}{2}}{sin\ \frac{\theta}{2}} \qquad (6)$$

The angle of minimum deviation δ is obtained from the difference between the angles ϕ_1 and ϕ_2 measured at the two different prism position (Fig. 6):

$$\delta = \frac{\varphi_1 - \varphi2}{2}$$

The dispersion curve (Fig. 4) is determined from the angles measured for the various mercury spectral lines.

The performance of a spectrometer is characterised by its 'resolving power'. Two wavelengths λ and $\lambda + d\lambda$ are still perceived as separate spectral lines when the principal maximum of line $\lambda + d\lambda$ coincides with the minimum of line λ.
The resolving power R is generally defined by the expression:

$$R = \frac{\lambda}{d\lambda}$$

For a prism, the following applies:

$$R = b \cdot |\frac{dn}{d\lambda}|$$

where b is the base of the prism (see Fig. 5).

Resolving power R is determined in the 'yellow' and the 'blue' regions of the spectrum (Table 1) from the slope of the dispersion curve (Fig. 4) with the prism fully illuminated ($b = 30\ mm$).

Spectral region: yellow	$\frac{dn}{d\lambda}$/cm^{-1}	$\frac{\lambda}{d\lambda}$
Flint glass	691	2073
Crown glass	377	1131

Spectral region: blue	$\frac{dn}{d\lambda}$/cm^{-1}	$\frac{\lambda}{d\lambda}$
Flint glass	2365	7095
Crown glass	1126	3378

Table 1: The dispersions and resolving powers of glass prisms determined from the dispersion curve (Fig. 4)

Example:

A prism with the resolving power

$$R = \frac{\lambda}{d\lambda} = 1000$$

is still able to separate the two sodium-D lines.

Dispersion and resolving power of a grating

If monochromatic light with the wavelength λ impinges on a diffraction grating, the intensity diffracted according to the angle φ is given by:

$$I(\varphi) = I(0) \cdot I(\varphi) = I(0) = (\frac{sin\ u}{u})^2 (\frac{sin\ N\ v}{sin\ v})^2;$$

with $v = \pi \frac{g}{\lambda} sin\ \varphi$ and $u = \pi \frac{s}{\lambda} sin\ \varphi$ \qquad (7)

(s = width of the slit; g = distance between two slits = grating constant; N = number of slits)

The first bracket describes the distribution of intensities due to diffraction by a single slit, whereas the combined effect of all the slits is described by the second bracket. If one bracket is zero, then total intensity I(φ) = 0. This means however, that minima due to a single slit continue to exist when N slits act together. On the other hand, peaks due to a single slit can be interspersed by further secondary minima if the second bracket is zero.
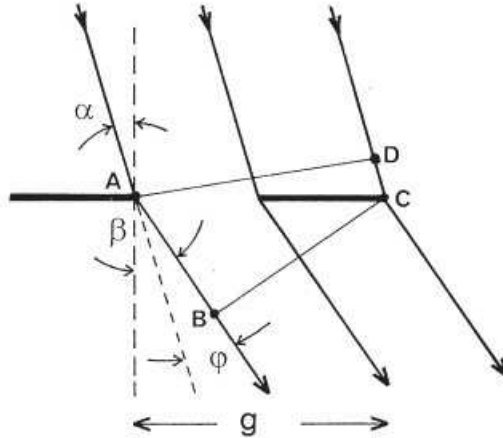

Fig. 7: Diffraction at the grating.

$$AB - CD = g(sin\ \beta\ -\ sin\ \alpha)\ =\ z\ \lambda$$

The diffraction direction φ of maximum z for a given grating fulfils the following relation:

$$v_z\ =\ \pi\,\frac{g}{\lambda}\,sin\ \varphi_z\ =\ z\,\pi \text{ or } sin\ \varphi_z\ =\ z\,\frac{\lambda}{g};$$

z = order of diffraction = $\pm$ (0,1,2,...).   (8)

There are (N-1) secondary minima between every two peaks. If light impinges at an angle α against the perpendicular to the grating, the following is valid (cf. Fig. 7):

$$sin\ \varphi + sin\ \beta\ =\ \frac{z\,\lambda}{g}\ =\ 2\,sin\frac{\alpha+\beta}{2}\cdot cos\frac{\alpha-\beta}{2}\ =\ 2\,sin\frac{\varphi}{2}\,cos\frac{\alpha-\beta}{2};$$

with $\beta\ -\ \alpha\ =\ \varphi$.   (9)

The angle β is considered to be positive when the diffracted and incident beams are on the same side of the perpendicular to the grating. If each angle is on another side of the perpendicular, then β is negative. In case of perpendicular incidence ($\alpha\ =\ 0$), the following applies:

$$sin\ \varphi\ =\ z\,\lambda\ /\ g.$$ (10)

By differentiation of (9) one obtains the angle of dispersion dφ/dλ of the grating:

$$\frac{d\varphi}{d\lambda}\ =\ \frac{z}{g\,cos\,\beta}\ =\ \frac{z}{g\,cos\,\varphi} \text{ (with } \beta\ =\ \varphi \text{, if } \alpha\ =\ 0\text{).}$$ (11)

Angular dispersion is independent of the angle of incidence and compared to prisms, it remains nearly constant for small diffraction angles.

Two spectral lines λ_1 and λ_2 only can be separated if they are so far apart from each other that the peak of λ_1 coincides with the minimum of λ_2 (Rayleigh criterion). The quotient of the average wavelength and the difference between the wavelengths of lines which merely appear separated is called the spectral resolving power.

$$A\ =\ \frac{\frac{1}{2}\,(\lambda_1\ +\ \lambda_2)}{(\lambda_2\ -\ \lambda_1)}\ =\ \frac{\bar{\lambda}}{\Delta\lambda}.$$ (12)

The following is valid for the spectral resolving power of a diffraction grating:

$$A\ =\ z\cdot N.$$ (13)

(z = order of diffraction; N = effective (illuminated) number of slits)

PHYWE excellence in science

Robert-Bosch-Breite 10
D - 37079 Göttingen

Tel: +49 551 604 - 0
Fax: +49 551 604 - 107

info@phywe.de
www.phywe.com

No.	Colour	λ/nm	φ(1)	φ(2)	φ(1)	φ(2)	φ̄	g/µm
			z = +1		z = -1			
1	viol.	404.656	253° 56'	73° 56'	225° 40'	45° 43'	14.12°	1.6586
2	blue	435.405	255° 05'	75° 06'	224° 35'	44° 38'	15.24°	1.6562
3	bl.-green	491.604	257° 06'	77° 07'	222° 34'	42° 39'	17.25°	1.6578
4	green	546.074	259° 06'	79° 08'	220° 37'	40° 41'	19.23°	1.6577
5	yellow	576.960	260° 15'	80° 16'	219° 28'	39° 34'	20.37°	1.6575
6	yellow	578.966	260° 20'	80° 20'	219° 23'	39° 27'	20.46°	1.6565

Table 2: Typical measurement values (diffraction of the first order) to determine the grating constant of the Rowland grating.

Table 2 shows the results (for $z = \pm 1$) of a typical measurement for the determination of grating constant g according to (10). The diffraction angle φ of a spectral line is calculated from the half angular difference of the corresponding diffraction order $\pm z$. Finally, as two values $\varphi(1)$ and $\varphi(2)$ are determined for every angle due to the two verniers, the average value is:

$$\bar{\varphi} = 1/4 \left[(\varphi(1)_{+z} - \varphi(1)_{-z}) - (\varphi(2)_{+z} - \varphi(2)_{-z}) \right].$$

If one also considers the refraction of lines No.1-No. 6 in the second order ($z = \pm 2$), as well as those third order lines ($z = \pm 3$) which still can be recognised, the grating constant is found to be:

g = (1.6567 ± 0.0016) µm ; $\Delta g/g$ = ± 0,1 %; (603.6 ± 0.6)/mm.

Table 3 shows the angular dispersion values determined according to (11) ($z = \pm 1$; d must be converted to radian for assessment. The values dφ and dλ are obtained from the difference between the corresponding values of neighbouring lines).

No.	λ/nm	φ/°	dλ/nm	dφ/rad	(dφ/dλ)m^{-1}	(g cosφ)$^{-1}$/m^{-1}
1	404.656	14.12				
2	435.405	15.24	30.75	0.01955	6.36×10^5	6.22×10^5
3	491.604	17.25	56.20	0.03490	6.21×10^5	6.28×10^5
4	546.074	19.23	54.50	0.03490	6.40×10^5	6.35×10^5
5	576.960	20.37	30.90	0.01989	6.44×10^5	6.41×10^5
6	578.966	20.46	2.00	0.00152	7.56×10^5	6.44×10^5

Table 3: Evaluation of the angular dispersion.

According to (12), a theoretical resolving power A = 562 nm/ 32 nm = ~17.6 is required to separate the green Hg-line-λ_1 ~546 nm from the pair of yellow lines λ_2 ~ 578 nm. To separate the two yellow Hg-lines λ_1 = 576.960 nm and λ_2 = 578.966 nm, A must at least be 289.

Table 4 gives the averages of the auxiliary slit width x obtained over several measurements, which are necessary to separate the lines of different diffraction orders z for various gratings. The last three columns give the values for the corresponding resolving power A.

The average values of A required to separate the yellow-green as well as the yellow-yellow lines, which are determined experimentally to be 17.7 and 287, concur sastisfactorily with the theoretical values.