Duane-Hunt displacement law and Planck's quantum of action (Item No.: P2540901)

Curricular Relevance

| Area of Expertise: ILIAS | Education Level: Physik | Topic: Hochschule | Subtopic: Moderne Physik | Experiment: Röntgenphysik |

Difficulty	**Preparation Time**	**Execution Time**	**Recommended Group Size**
Difficult	1 Hour	2 Hours	2 Students

Additional Requirements:

- PC

Experiment Variations:

Keywords:

X-ray tubes, bremsstrahlung, characteristic X-radiation, energy levels, crystal structures, lattice constant, interference, Bragg's law

Overview

Short description

Principle

The experiment is about the determination of the boundary wavelength of the bremsspectrum of a copper anode that decreases when the anode voltage increases. The short-wave onset of the bremsspectrum enables the verification of Duane-Hunt's displacement law and the determination of Planck's quantum of action.

This experiment is included in the "XRP 4.0 X-ray solid state", "XRS 4.0 X-ray structural analysis", and "XRC 4.0 X-ray characteristics" upgrade sets.

Alternatively, a tungsten tube (09057-80) can be used for this experiment.

Fig. 1: P2540901

Equipment

Position No.	Material	Order No.	Quantity
1	XR 4.0 expert unit, X-ray unit, 35 kV	09057-99	1
2	XR 4.0 X-ray goniometer	09057-10	1
3	XR 4.0 X-ray Plug-in Cu tube	09057-51	1
4	Geiger-Mueller counter tube, 15 mm (type B)	09005-00	1
5	XR 4.0 X-ray LiF crystal, mounted	09056-05	1
6	XR 4.0 Software measure X-ray	14414-61	1
7	Data cable USB, plug type A/B, 1.8 m	14608-00	1
8	XR 4.0 X-ray Diaphragm tube d = 2 mm	09057-02	1

Tasks

1. Record the X-ray spectrum that is emitted by the copper anode at various different anode voltages U_A as a function of the Bragg angle ϑ and with the aid of a **LiF** monocrystal as the analyser.

2. Determine the short-wave onset (λ_{min}) of the bremsspectra.

3. Use the results to verify the Duane-Hunt displacement law, and to determine Planck's "quantum of action".

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Set-up and procedure

Set-up

Connect the goniometer and the Geiger-Müller counter tube to their respective sockets in the experiment chamber (see the red markings in Fig. 2). The goniometer block with the analyser crystal should be located at the end position on the right-hand side.

Fasten the Geiger-Müller counter tube with its holder to the back stop of the guide rails. Do not forget to install the diaphragm in front of the counter tube (see Fig. 3).

Insert a diaphragm tube with a diameter of $2\,mm$ into the beam outlet of the tube plug-in unit for the collimation of the X-ray beam.

For calibration: Make sure, that the correct crystal is entered in the goniometer parameters. Then, select "Menu", "Goniometer", "Autocalibration". The device now determines the optimal positions of the crystal and the goniometer to each other and then the positions of the peaks

Fig. 2: Connectors in the experiment chamber
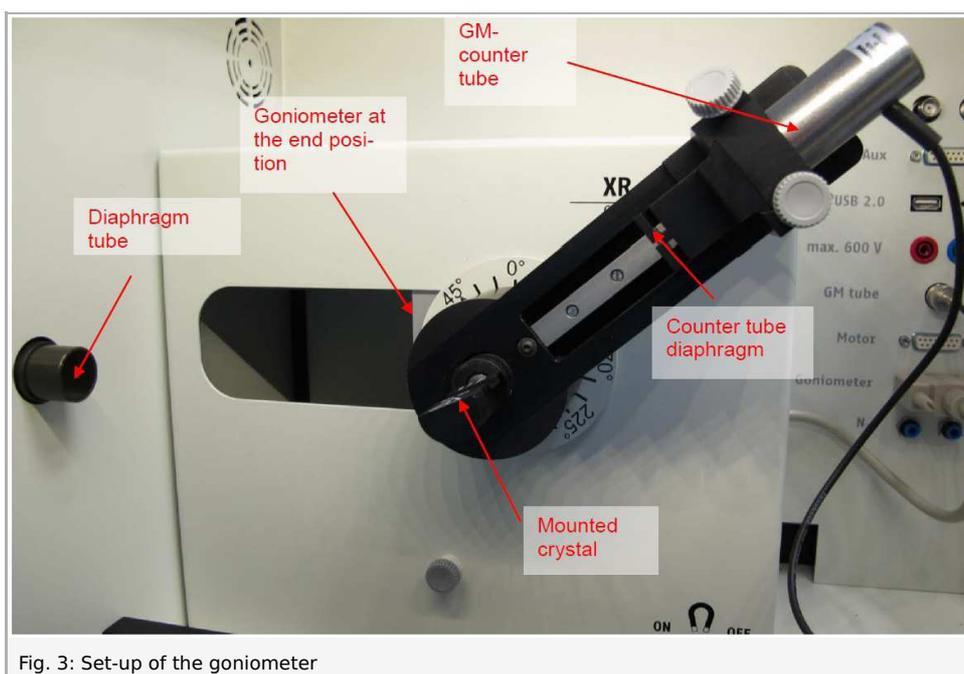
Fig. 3: Set-up of the goniometer

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Note

Details concerning the operation of the X-ray unit and goniometer as well as information on how to handle the monocrystals can be found in the respective operating instructions.

Procedure

- Connect the X-ray unit via USB cable to the USB port of your computer (the correct port of the X-ray unit is marked in Fig. 4).

Fig. 4: Connection of the computer

- Start the "measure" program. A virtual X-ray unit will be displayed on the screen (Fig. 5).

- You can control the X-ray unit by clicking the various features on and under the virtual X-ray unit. Alternatively, you can also change the parameters at the real X-ray unit. The program will automatically adopt the settings.

- Click the experiment chamber (see the red marking in Figure 5) to change the parameters for the experiment (e.g. the goniometer).

- If you click the X-ray tube (see the red marking in Figure 5), you can change the voltage and current of the X-ray tube. First record an entire spectrum ($4°$- $55°$). Then, record the spectra up to the K_β line (scanning range $4°$- $22°$) with an anode current of $1\,mA$ and a voltage range of 13- $33\,kV$ in steps of $2\,kV$.

- Start the measurement by clicking the red circle:
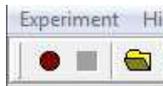
- After the measurement, the following window appears:
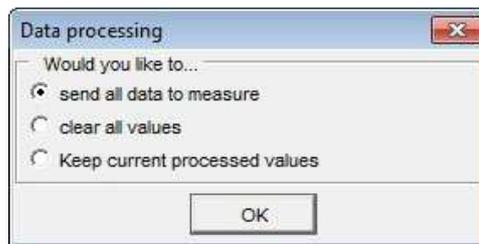
- Select the first item and confirm by clicking OK. The measured values will now be transferred directly to the "measure" software.

- At the end of this manual a short introduction to the evaluation of data using the program "measure" is given.

Note

Never expose the Geiger-Müller counter tube to the primary X-radiation for an extended period of time.

Fig. 5: Part of the user interface of the software

Overview of the settings of the goniometer and X-ray unit:

- Auto and $2:1$ coupling mode
- Gate time $2\,s$; angle step width $0.1°$

Recording of the entire spectrum:

- Scanning range $3°$- $55°$
- Anode voltage $U_A = 35\,kV$; anode curre
  $I_A = 1\,mA$

Bremsspectra up to the characteristic Cu K_β line ($n = 1$) as a function of U_A:

- Scanning range $4°$- $22°$
- Anode voltage
  $33\,kV > U_A > 13\,kV$, $\Delta U_A = 2kV$; anode
  current $I_A = 1\,mA$

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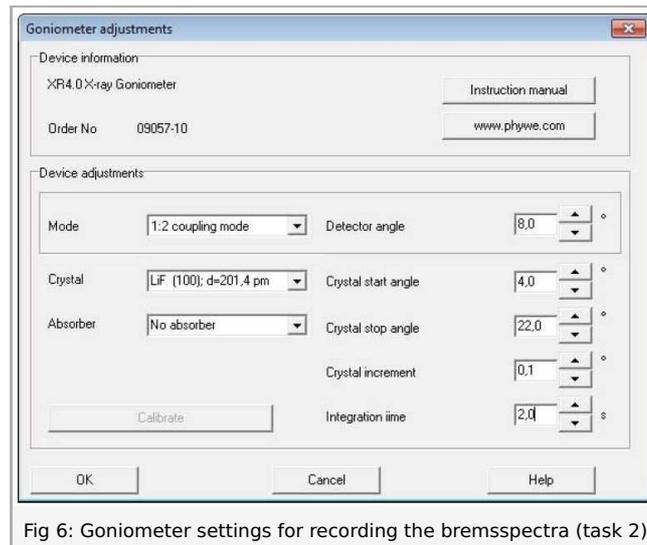
Fig 6: Goniometer settings for recording the bremsspectra (task 2)

Theory and evaluation

Theory

Due to the voltage U_A between the anode and cathode, the electrons are accelerated from the cathode towards the anode. At the cathode, the electrons have the following energy:

$$E_{kin} = e \cdot U_A \quad (e = \text{elementary charge}) \quad\quad (1)$$

Due to interactions with the atoms of the anode material, the electrons gradually lose their kinetic energy, which is converted into a continuous X-ray spectrum (bremsspectrum). If the kinetic energy is
lost in one step, X-rays with maximum energy (minimum wavelength λ_{min}) are generated. In 1915, the American physicists Duane and Hunt found that the product of the accelerating voltage, and of the minimum wavelength, is constant:

$$U_A \cdot \lambda_{min} \propto 1.25 \cdot 10^{-6} \, V \cdot m \quad\quad (2)$$

The energy equation:

$$E_{kin} = e \cdot U_A = h \cdot f_{max} = h \frac{c}{\lambda_{min}} \qu\quad (3)$$

Planck's constant	h	$= 6.6256 \cdot 10^{-34} \, Js$
Velocity of light	c	$= 2.9979 \cdot 10^8 \, m/s$
Equivalent	$1 \, eV$	$= 1.6021 \cdot 10^{-19} \, J$

leads to:

$$\lambda_{min} = 1.2398 \cdot 10^{-6} \frac{1}{U_A} V \cdot m$$

for the shortest wavelength of the X-ray photons.

A **LiF** monocrystal is used for the wavelength analysis of the X-rays. When the X-rays impinge on the lattice planes of the monocrystal under the glancing angle ϑ, the rays that are reflected interfere with each other in a constructive manner provided that their path difference corresponds to an integral multiple of the wavelength. In this case, Bragg's law applies:

$$2d \sin \vartheta = n\lambda \qu\quad\quad (4)$$

(**LiF** (200) interplanar spacing $d = 201.4 \, pm$; $n = 1, 2, 3, \ldots$)

Evaluation

Task 1: Record the X-ray spectrum that is emitted by the copper anode at various different anode voltages U_A as a function of the Bragg angle ϑ and with the aid of a LiF monocrystal as the analyser.

Figure 7 shows the entire bremsspectrum of the copper anode. Figure 8 only shows the section that is of interest for further evaluation, based on three different anode voltages. When the anode voltage is
increased, the onset of the bremsspectrum is shifted towards smaller glancing angles, i.e. towards shorter wavelengths.

Fig. 7: X-ray spectrum of copper; LiF analyser crystal
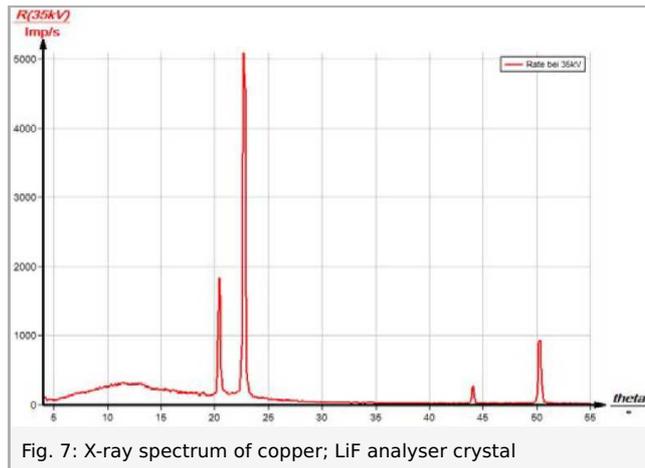
Task 2: Determine the short-wave onset (λ_{min}) of the bremsspectra.

The short-wave limit λmin of the bremsspectrum is determined by the associated glancing angle and it can be calculated with the aid of equation (4).

Fig. 8: Bremsspectrum of copper for three different anode voltages $U_A(15\,kV,\,25\,kV,$ and $31\,kV)$, x-axis: glancing angle $\vartheta/°$
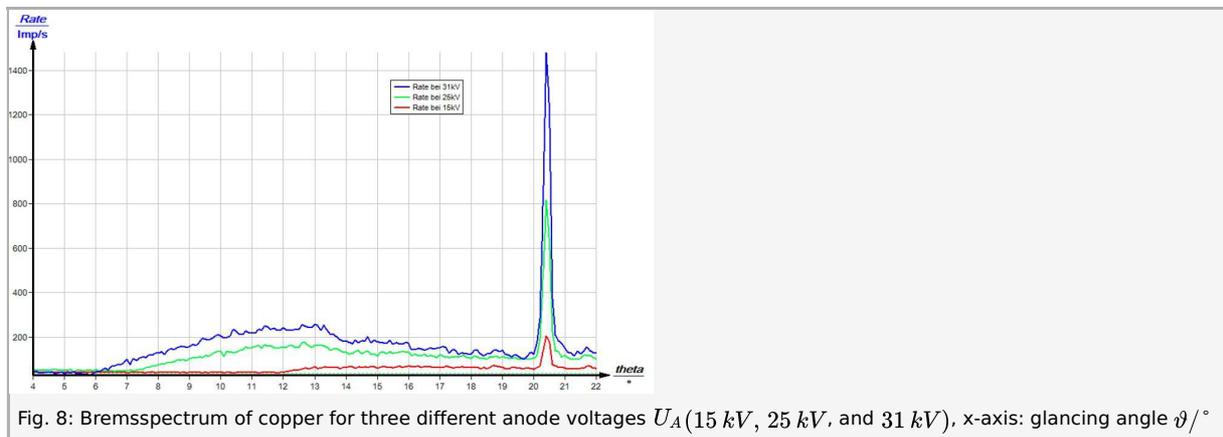
Task 3: Plot the functions $\lambda_{min} = f(1/U_A)$ and $\sin\vartheta_{min} = f(1/U_A)$. Calculate Planck's quantum of action.

Figure 9 shows the λ_{min} values that were determined based on the bremsspectra as a function of $1/U_A$.

Fig. 9: Duane-Hunt displacement law; $\lambda = f(1/U_A)$
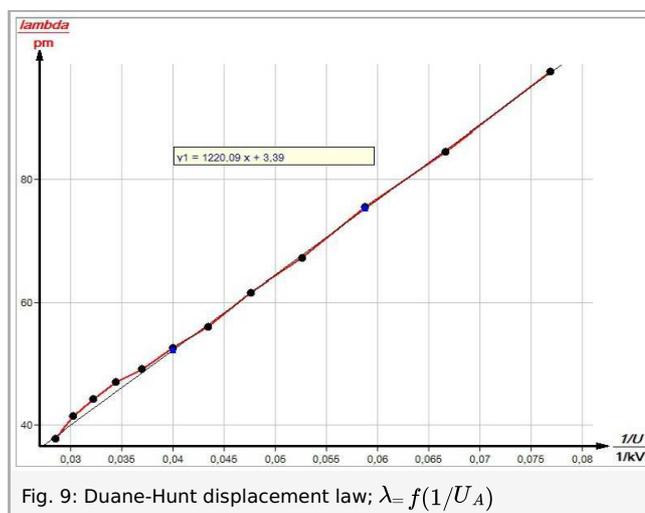
The value of the gradient m of the resulting straight line confirms the relationship in accordance with Duane-Hunt's law of

displacement, equation (2):

$$m = \frac{\lambda_{min}}{1/U_A} = (12.20 \pm 0.07) \cdot 10^{-5}\, V \cdot m$$

Planck's quantum of action can be determined with the aid of the family of bremsspectra. (3) and (4) lead to:

$$U_A = \frac{h \cdot c}{2e \cdot d \cdot \sin \vartheta} \qquad\qquad\qquad (5)$$

If $\sin \vartheta_{min}$ is plotted as a function of $1/U_A$, a straight line results (see Figure 11). The gradient of this straight line can be obtained from Figure 10:

$$m = \frac{h \cdot c}{2e \cdot d}$$

With the experimental value of $m = (2.986) \cdot 10^3\, pm \cdot V$, the following results for Planck's quantum of action:

$$h = \frac{m \cdot 2e \cdot d}{c} = 6.43 \cdot 10^{-34}\, Js;\ \frac{\Delta h}{h} = \pm\, 3\,\%$$

Fig. 10: Planck's quantum of action: $\sin \vartheta = f(1/U_A)$

Fig. 11: Duane-Hunt line with the automatic calculation of the Planck's constant
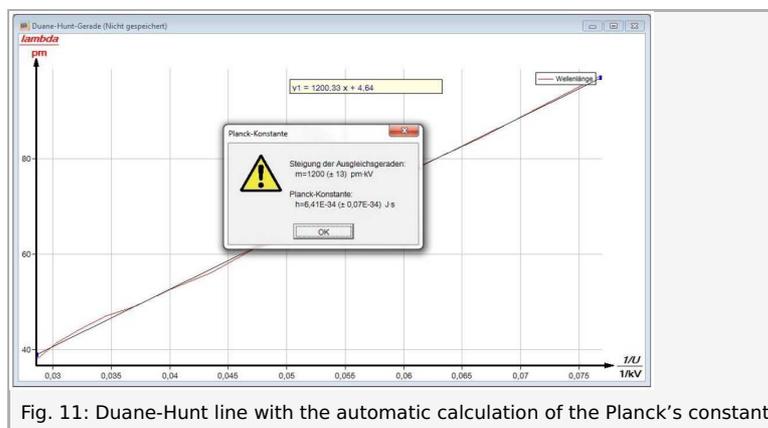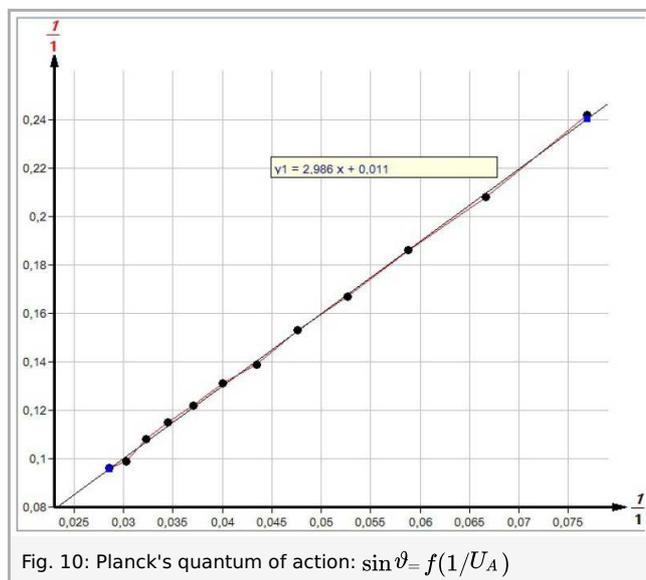
Evaluation of the measurements with the aid of the "measure" software

In order to evaluate the measurements with the aid of the "measure" software, first convert the glancing angles ϑ (crystal angle = x-axis) of the spectra into the corresponding wavelengths. To do so, proceed as follows:

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1. "Analysis", "X-ray spectroscopy", "Convert x-axis", and "Wavelength (calculate)".

The Duane-Hunt line can be obtained from the converted spectra (Imp/s = f(lamdba/pm) as follows:

2. Click "Analysis", "X-ray spectroscopy", and "Duane-Hunt straight line". The window "Duane-Hunt straight line" appears.

   Then, select the onset of the bremsspectrum as a narrow band with the aid of the marker and click "Accept value". The additional window then shows the corresponding values for the anode voltage and wavelength. Repeat this for the other spectra with different anode voltages.

   In order to generate the straight line, click "Generate Duane-Hunt line".

   Planck's quantum of action can be determined by clicking "Analysis", "X-ray spectroscopy", and "Determine Planck constant" (see Fig. 11).

   With "Display options", "Channels", and "Symbol", the measuring points of the Duane-Hunt line can be displayed if desired.

Refer to the Help of the "measure" software for additional, more detailed explanations concerning the program features.