Equation of state of ideal gases with Cobra4

(Item No.: P2320162)

Curricular Relevance
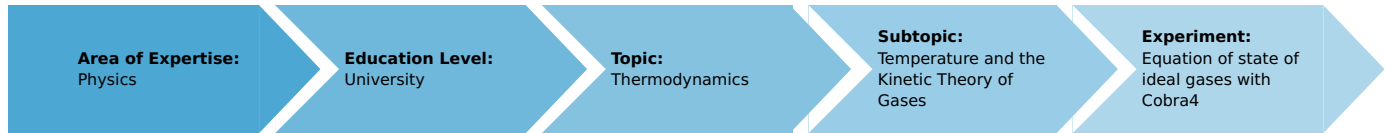
| Area of Expertise: Physics | Education Level: University | Topic: Thermodynamics | Subtopic: Temperature and the Kinetic Theory of Gases | Experiment: Equation of state of ideal gases with Cobra4 |

Difficulty	**Preparation Time**	**Execution Time**	**Recommended Group Size**
Intermediate	1 Hour	1 Hour	2 Students

Additional Requirements:

- PC with USB interface, Windows XP or higher
- Software measureLAB

Experiment Variations:

Keywords:

Pressure, Temperature, Volume, Cubic compressibility coefficient, Thermal tension coefficient, Coefficient of thermal expansion, Ideal gas law, Universal gas constant, Boyle and Mariotte's law, Charles' (Amontons') law, Gay-Lussac's law

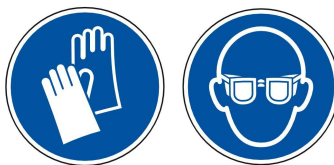# Overview

Short description

Principle

The state of a gas is determined by temperature, pressure and amount of substance. For the limiting case of ideal gases, these state variables are linked via the ideal gas law. For a change of state under isobaric conditions this equation converts to Gay-Lussac's first law while under isochoric conditions it becomes Amontons' and in the case of isothermal process control it converts to Boyle and Mariotte's law.

Fig. 1: Experimental set-up.

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Safety instructions

When handling chemicals, you should wear suitable protective gloves, safety goggles, and suitable clothing.

Equipment

Position No.	Material	Order No.	Quantity
1	Set Gas laws w. glass jacket & Cobra4	43021-00	1
2	Power regulator	32288-93	1
Additional material			
	PC, Windows® XP or higher		

Tasks

1. Experimentally investigate the validity of the three gas laws for a constant amount of gas (air).
2. Calculate the universal gas constant from the relationship obtained.
3. Calculate the thermal coefficient of expansion from the results of measurements under isobaric conditions.
4. Calculate the thermal coefficient of tension from the results of measurements under isochoric conditions.

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Setup and Procedure

- Set up the experiment as shown in Fig. 1.
- Connect the Cobra4 Sensor-Unit Thermodynamics with the Cobra4 MobileLink2. Connect the thermocouple with the T1 socket of the Cobra4 Sensor-Unit Thermodynamics.
  Start the PC and connect it with the Cobra4 MobileLink2.

  Start the software "measureLAB" 🟥 on your computer and choose the experiment from the start screen ("PHYWE experiments", search for "P2320162", and click on the folder that contains this experiment). All necessary presettings will be loaded.

  After the Cobra4 Mobile-Link has been switched on, the sensor is automatically recognized.
- Install the gas syringe in the glass jacket as described in the operating instructions supplied with the glass jacket. Pay particular attention to the air-tightness.
- As an exception here, because no air is to be allowed to leak out even at higher pressures, lubricate the plunger with a few drops of multigrade motor oil, so that the glass plunger is covered with an uninterrupted clear film of oil throughout the entire experiment; but avoid excess oil.
- Fill the glass jacket with water via the funnel and insert a magnetic stirrer bar.
- Connect a silicone tube to the hose nipple of the jacket's upper tubular sleeve so that the bath fluid which expands on heating can flow through the tube into a beaker.
- Insert the thermocouple and place it as close to the syringe as possible.
- After adjusting the initial volume of the gas syringe to exactly 50 ml, connect the nozzle of the gas syringe to Cobra4 Sensor-Unit Thermodynamics via a short piece of rubber tubing. Keep the tubing connections as short as possible.
- Secure the tubing on both the gas syringe's nozzle and on the reducing adapter with hose clips.

Procedure

1. Boyle and Mariotte's law

-
- Start the measurement with .
- Subsequently expand the enclosed quantity of air in 1 ml steps to a volume of approximately 65 ml.
- Record the volume for each step by clicking on .
- Mix the water in the glass jacket by moving the magnetic stirrer bar with the aid of a bar magnet and facilitate pressure equilibration in the gas syringe by turning the plunger.
- Terminate the measurement by pressing .
- After termination, the measureLAB software presents a graph that shows the correlation between volume and pressure as constant temperature (Fig. 3).
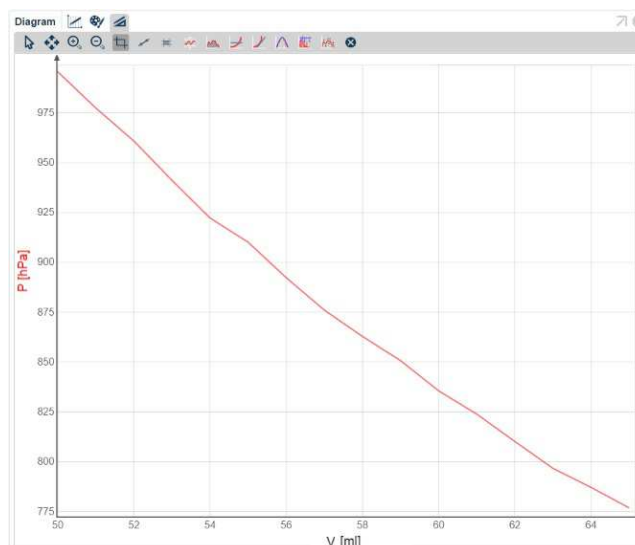

Fig. 3: Correlation between the volume V and the pressure p at constant temperatures(T = 295.15 K) and constant quantity of substance (n = 2.086 mmol).

- To have the plot of pressure versus the reciprocal volume, click on the symbol 🟩 to open the data pool (Fig. 4).

Fig. 4: Data pool

Now you can perform some channel modifications by clicking on √a̅. First, drag and drop your measure data (volume) to measurements, then drag and drop the data into your formula (see Fig. 5).
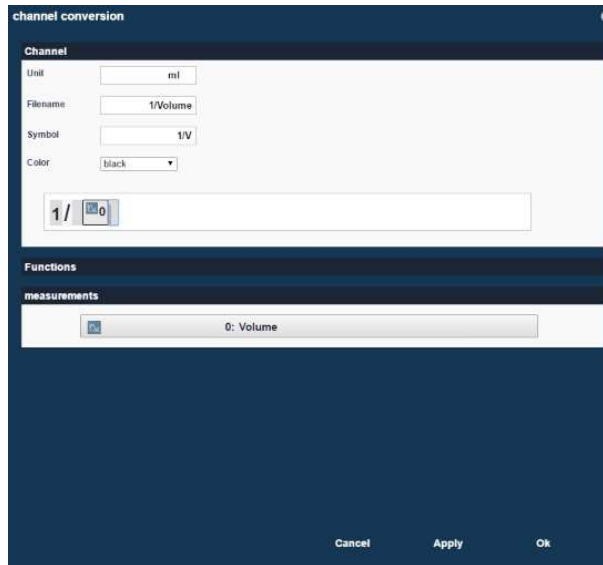

Fig. 5: Settings for channel conversion

- Go back to data pool and select the measure data for pressure and your modified channel pVT. When selected, choose the option 'Diagram' and the software will present you the desired graph as can be seen in Fig. 6 that shows the correlation between pressure p and the quantity $1/V$.
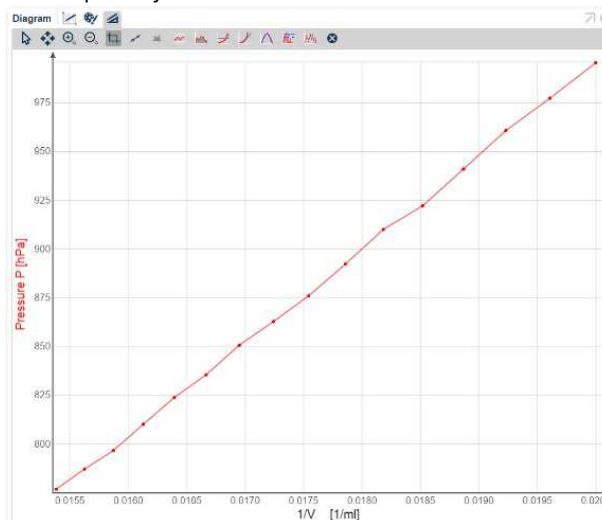

Fig. 6: Pressure p as a function of the reciprocal volume $1/V$ at constant temperature (T = 295.15 K) and constant quantity of substance (n = 2.086 mmol).

- With you can let the program show the slope.

2. Gay-Lussac's law

-
- Start the measurement with .
- Record the first value for the initial temperature by clicking .
- Switch on the heating apparatus and adjust the power regulator so that the glass jacket is slowly heated.
- Mix the water in the glass jacket by moving the magnetic stirrer bar with the aid of a bar magnet and facilitate pressure

equilibration in the gas syringe by turning the plunger.

- After each 1 ml increase in volume, take the next value.
- After the gas volume has reached 60 ml, switch off the heating apparatus and terminate the measurement by pressing .
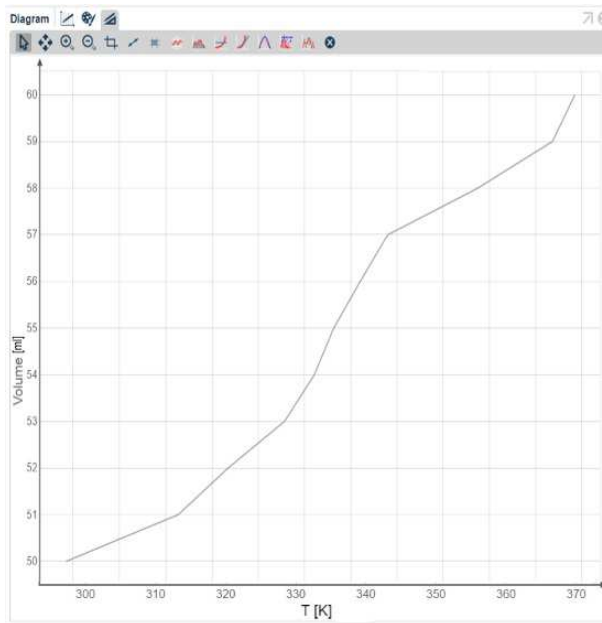- Fig. 7 shows the graph as it is then presented by the program.

Fig. 7: Dependence of the volume V on the temperature T at constant pressure p = 1020 hPa and constant amount of substance (n = 2.23 mmol).

- To have the plot of the quantity *pV/T* versus volume, go to data pool ⬢ and click on √α .

  Now you can perform some channel modifications, drag and drop the measure data for volume, temperature and pressure to measurements. Subsequently, drag and drop the data to the formula (see Fig. 8).

Fig. 8: Settings for channel modification.

- Go to data pool ⬢ and select the measure data for volume and your modified channel pVT. When selected, choose the option 'Diagram' and the software will present you the desired graph, as can be seen in Fig. 9 (the dependence of the quantity pV/T on the volume V).
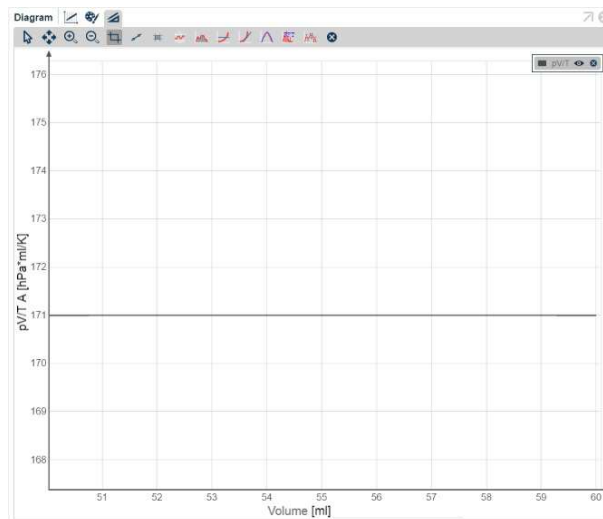
Fig. 9: The quantity pV/T depending on the volume V.

3. Amontons' law

-
- Start the measurement with .
- Subsequently, adjust the heating apparatus to slow heating with the power regulator.
- Mix the water in the glass jacket by moving the magnetic stirrer bar with the aid of a bar magnet and facilitate pressure equilibration in the gas syringe by turning the plunger.
- Record the pressure corresponding to the initial temperature by clicking on .
- After each temperature increase of 5 K, push the plunger rapidly into the gas syringe until the gas volume is compressed to the initial volume of V = 50 ml and take the next value by clicking on .
- After the temperature has reached approximately 370 K or if there is an evident loss of air during compression, switch off the heating apparatus and terminate the measurement by pressing .
- Fig. 13 shows the graph for the dependence of the pressure p on the temperature T at constant volume V as it is then presented by the program.

Fig. 10: Dependence of the pressure p on the temperature T at constant volume (V = 50 ml) and constant amount of substance (n = 2.23 mmol).

- To have the plot of the quantity pV/T versus temperature, go to data pool and click on .

  Now you can perform some channel modifications, drag and drop the measure data for volume, temperature and pressure to measurements. Subsequently, drag and drop the data to the formula (see Fig. 8).
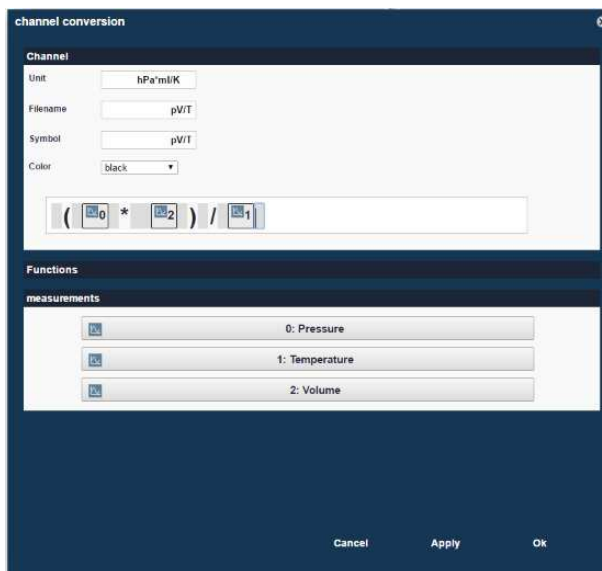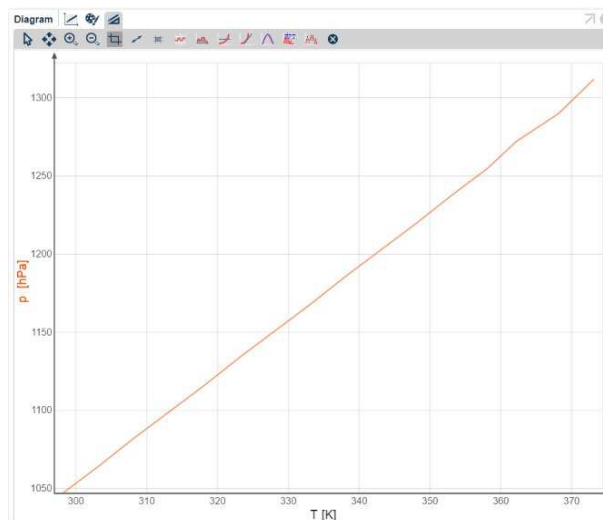
- Go to data pool and select the measure data for temperature and your modified channel pVT. When selected, choose the option 'Diagram' and the software will present you the desired graph, as can be seen in Fig. 11.

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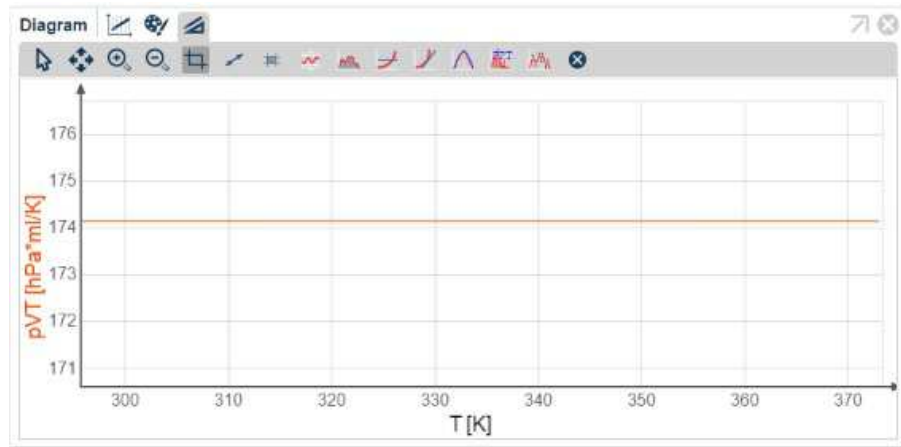
Fig. 15: The quantity pV/T depending on the temperature T.

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Theory and evaluation

The state of a gas is a function of the state variables temperature T, pressure p and the amount of substance n, which reciprocally determine one another. Thus, the dependence of pressure on the temperature, volume and amount of substance variables is described by the total differential

$$\mathrm{d}V = \left(\frac{\partial V}{\partial T}\right)_{p,n} \mathrm{d}T + \left(\frac{\partial V}{s\partial p}\right)_{T,n} \mathrm{d}p + \left(\frac{\partial V}{\partial n}\right)_{T,V} \mathrm{d}n \text{ (1.1)}$$

Analogously, the following is true for the change of pressure with T, V and n:

$$\mathrm{d}p = \left(\frac{\partial p}{\partial T}\right)_{V,n} \mathrm{d}T + \left(\frac{\partial p}{\partial V}\right)_{T,n} \mathrm{d}V + \left(\frac{\partial p}{\partial n}\right)_{T,V} \mathrm{d}n \text{ (1.2)}$$

This relationship simplifies for a given amount of substance (n = const., $\mathrm{d}n = 0$; enclosed quantity of gas in the gas syringe) and isothermal change of state (T = const., $\mathrm{d}T = 0$ >) to

$$\mathrm{d}V = \left(\frac{\partial V}{\partial T}\right)_{T,n} \mathrm{d}p \text{ (2.1)}$$

and

$$\mathrm{d}p = \left(\frac{\partial p}{\partial T}\right)_{V,n} \mathrm{d}T \text{ (2.2)}$$

The partial differential quotient $(\partial V / \partial p)\, T, n$ resp. $(\partial p / \partial V)\, T, n$ corresponds geometrically to the slope of a tangent to the function $V = f(p)$ or $p = f(V)$ and therefore characterises the mutual dependence of pressure and volume. The degree of this dependence is determined by the initial volume or the initial pressure. One thus defines the cubic compressibility coefficient X_0 by referring it to V or V_0 at $T_0 = 273.15 \text{ K}$.

$$X_0 = \frac{1}{V_0} \left(\frac{\partial V}{\partial p}\right)_{T,n} \text{ (3)}$$

The partial differential quotient $(\partial p / \partial T)\, V, n$ corresponds geometrically to the slope of a tangent to the function $p = f(T)$ and thus characterises the dependence of the pressure on the temperature. The degree of this dependence is determined by the initial pressure. Therefore, one defines the thermal coefficient of tension β_0 as a measure of the temperature dependence by referring it to p or p_0 at $T_0 = 273.15 \text{ K}$.

$$\beta_0 = \frac{1}{p_0} \left(\frac{\partial p}{\partial T}\right)_{V,n} \text{ (4)}$$

The partial differential quotient $(\partial V / \partial T)\, p, n$ corresponds geometrically to the slope of a tangent to the function $V = f(T)$ and thus characterises the mutual dependence of volume and temperature. The degree of this dependence is determined by the initial volume. The thermal coefficient of expansion y_0 is therefore defined as a measure of the temperature dependence of the volume by referring it to V or V_0 at $T_0 = 273.15 \text{ K}$.

$$y_0 = \frac{1}{V_0} \left(\frac{\partial V}{\partial T}\right)_{p,n} \text{ (5)}$$

For the limiting case of an ideal gas (sufficiently low pressures, sufficiently high temperatures), the correspondence between the state variables p, V, T and n is described by the ideal gas law:

$$pV = nRT \text{ (6)}$$

R Universal gas constant

For cases of constant quantity of substances and isothermal process control this equation changes into the following equations:

$$pV = \text{const. (6.1)}$$

and

$$p = \text{const.} \frac{1}{V} \text{ (6.2)}$$

According to this correlation, which was determined empirically by Boyle and Mariotte, a pressure increase is accompanied by a volume decrease and vice versa. The graphic representation of the functions $V = f(p)$ or $p = f(V)$ results in hyperbolas (Fig. 5). In contrast, plotting the pressure p against the reciprocal volume $1/V$ results in straight lines where $p = 0$ at $1/V = 0$ (Fig.

6). From the slope of these linear relationships,

$$\left(\frac{\partial p}{\partial v^{-1}}\right)_{T,n} = nRT \ (7)$$

it is possible to determine the gas constant R experimentally when the enclosed constant quantity of air n is known. This is equal to the quotient of the volume V and the molar volume V_m,

$$n = \frac{V}{V_{\mathrm{m}}} \quad (8)$$

which is $V_0 = 22.414 \, \mathrm{l \cdot mol^{-1}}$ at $T_0 = 273.15 \, \mathrm{K}$ and $p_0 = 1013.25 \, \mathrm{hPa}$ at standard conditions. A volume measured at p and T is therefore first reduced to these conditions using the relationship obtained from (6):

$$\frac{p_0 V_0}{T_0} = \frac{p_1 V_1}{T_1} = \frac{pV}{T} \ (9)$$

For the limiting case of an ideal gas (sufficiently low pressure, sufficiently high temperature), the integration of a differential equation resulting from (1.2) and (4), where $\beta_0 = \mathrm{const.}$, yields

$$\frac{p_0}{T_0} = \frac{p}{T} \ (10.1)$$

and

$$p = \mathrm{const.} \ T \ (10.2)$$

According to this correlation, which was discovered by Charles and Amontons, the graphic presentation of the pressure as a function of the temperature results in an ascending straight line (Fig. 13) where $p = 0$ at $T = 0$.

From (4) and the ideal gas law (6) the following is true for the slope of these linear relationships

$$\left(\frac{\partial p}{\partial T}\right)_{\mathrm{V,n}} = p_0 \beta_0 = \frac{nR}{V} \ (11)$$

From this, the thermal coefficient of tension β_0 and the universal gas constant R can be determined for a known initial pressure p_0 and a known quantity of substance n. The enclosed constant amount of substance n is equal to the quotient of the volume V and the molar volume V_{m}.

For the limiting case of an ideal gas (sufficiently low pressure, sufficiently high temperature), the integration of a differential equation resulting from (1.2) and (5), where $y_0 = \mathrm{constant}$, yields

$$\frac{V_0}{T_0} = \frac{V}{T} \ (12.1)$$

and

$$V = \mathrm{const.} \ T \ (12.2)$$

According to this correlation, which was discovered by Gay-Lussac, the graphic presentation of the volume as a function of the temperature provides a scending straight lines (Fig. 7) where $V = 0$ for $T = 0$. From (5) and the ideal gas law (6) the following is true for the slope of these linear relationships:

$$\left(\frac{\partial V}{\partial T}\right) = V_0 \gamma_0 = \frac{nR}{p} \ (13)$$

From this, the thermal coefficient of expansion γ_0 and the universal gas constant R are experimentally accessible for a known initial volume V_0 and a known amount of substance n.

Fig. 12 and 15 show the quantity pV/T appearing to be nearly constant.

Data and results

The theoretical values for an ideal gas are

$$R \ (\mathrm{lit.}) = 8.31441 \, \mathrm{Nm \cdot K^{-1} \cdot mol^{-1}} = \mathrm{J \cdot K^{-1} \cdot mol^{-1}}$$

$$\gamma_0 \ (\mathrm{lit.}) = 3.661 \cdot 10^{-3} \, \mathrm{K^{-1}}$$

$$\beta_0 \ (\mathrm{lit.}) = 3.661 \cdot 10 - 3 \, \mathrm{K^{-1}}$$

1. Boyle and Mariotte's law

Figs. 5 and 6 confirm the validity of Boyle and Mariotte's law. From the slope obtained for

$n = 2.086 \, \mathrm{mmol}$ and $T = 295.15 \, \mathrm{K}, (\partial p/\partial V^{-1})T, n = 4.6464 \, \mathrm{kPa}/m^{-3} = 4.6464 \, \mathrm{Nm}$ of the linearised correlation

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between p and $1/V$ (Fig. 6), the universal gas constant can be calculated to be $R = 7.547\,\mathrm{Nm}\cdot\mathrm{K}^-$.

The deviation from the literature value is due to the unavoidable lack of gas-tightness with increasing deviation from atmospheric pressure through compression or expansion, whereby the condition $\mathrm{d}n = 0$ is violated and the observed slope $(\partial p/\partial V^{-1})T$ is diminished in comparison with the value measurable with a constant quantity of substance.

2. Gay-Lussac's law

The investigation of the correlation between volume and temperature with a constant quantity of gas of $n = 2.23\,\mathrm{mmol}$, calculated according to the relations (8) and (9), confirms the validity of the Gay-Lussac's first law, with the linear relationship demonstrated in Fig. 12.

From the corresponding slope $(\partial V/\partial T)p, n = 0.18\,\mathrm{ml/K}$ and for the initial volume $V_0 = 50\,\mathrm{ml}$, the following values are obtained for the universal gas constant R and the coefficient of thermal expansion γ_0.

$R\,(\text{exp.}) = 8.07174\,\mathrm{Nm}\cdot\mathrm{K}^{-1}\cdot\mathrm{mol}^{-1}$

$\gamma_0\,(\text{exp.}) = 3.04\cdot 10^{-3}\,\mathrm{K}^{-1}$

3. Amontons' law

The investigation of the correlation between pressure and temperature with a constant quantity of gas of $n = 2.23\,\mathrm{mmol}$, calculated according to the relations (8) and (9), confirms the validity of the Charles' (Amontons') law with the linear relationship demonstrated in Fig. 13.

From the corresponding slope $(\partial p/\partial T)V, n = 3.72\,\mathrm{hPa/K}$ and for the initial pressure $p_0 = 1002.2\,\mathrm{hPa}$, the following values are obtained for the universal gas constant R and the coefficient of thermal tension β_0.

$R\,(\text{exp.}) = 8.34\,\mathrm{Nm}\cdot\mathrm{K}^{-1}\cdot\mathrm{mol}^{-1}$

$\beta_0\,(\text{exp.}) = 3.71\cdot 10^{-3}\,\mathrm{K}^{-1}$

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