Fresnel’s law, theory of reflection (Item No.: P2250305)

Curricular Relevance

Area of Expertise: ILIAS

Education Level: Physik

Topic: Hochschule

Subtopic: Licht und Optik

Experiment: Ausbreitung von Licht

Difficulty	Preparation Time	Execution Time	Recommended Group Size
Difficult	10 Minutes	20 Minutes	2 Students

Additional Requirements:

Experiment Variations:

Keywords:

Electromagnetic theory of light, reflection coefficient, reflection factor, Brewster’s law, law of refraction, polarisation, degree of polarisation

Introduction

Overview

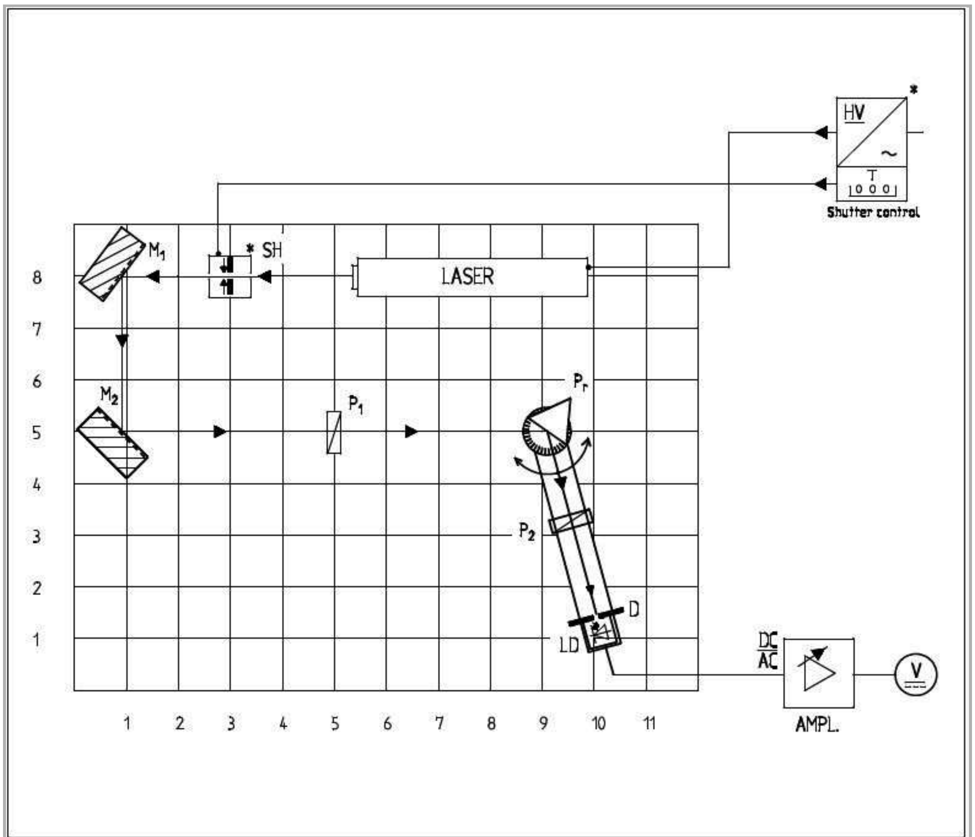

Fig. 1: Experimental set up for the verification of the rotation of the plane of polarisation due to reflection (* only required for 5 mW laser)

Equipment

Position No.	Material	Order No.	Quantity
1	Base plate with rubber feet	08700-00	1
2	HeNe laser	08180-93	1
3	Adjusting support 35 x 35 mm	08711-00	2
4	Surface mirror 30 x 30 mm	08711-01	2
5	Magnet foot	08710-00	5
6	Polarisation filter	08730-00	2
7	Prism table with support	08725-00	1
8	Prism, 60o, flint glass	08237-00	1
9	Rotating guide rail with angular scale and magnet foot	08717-00	1
10	Photocell, silicone	08734-00	1
11	Measurement amplifier, universal	13626-93	1
12	Voltmeter 0.3... 300 V/10 ... 300 V~	07035-00	1
13	Connecting cable, red, l = 500 mm	07361-01	2

Tasks

1. The reflection coefficient of light, which is polarised either perpendicularly or parallel to the plane of incidence, is to be determined as a function of the angle of incidence and plotted graphically
2. The refraction index of the flint glass prism is to be determined.
3. The refraction coefficient is to be calculated by means of Fresnel’s formula and compared to the measured curve.
4. The reflection factor for flint glass is calculated.
5. The rotation of the plane of polarisation for linearly polarised light after reflection is determined as a function of the angle of incidence and plotted graphically. This is compared to the values calculated by means of Fresnel’s formula.

Set-up and procedure

- The experimental set up is shown in fig. 1. The recommended set up height (height of beam path) should be 130 mm.
- *Set up of the rotating unit*: To start with, the stopping screw of a magnet foot is removed. The circular orifice of the rotating guide rail is set under the foot. The angular scale is set onto the magnet foot and also on top of the rotating guide rail. The magnet foot is fixed to the optical base plate, and the rotating guide rail can be shifted sufficiently. Photocell **LD** can be fixed to one end and polarisation filter P_2 to the middle of the rotating guide rail, both by means of a magnet foot. During the set up of the optical base plate, the angular distribution should be reasonable, that is, the 0° scale mark should be directed towards the incident laser beam.
- Prism **Pr** should be placed with the forward surface edge exactly on the central point of the table. The laser beam is then adjusted onto the central axis of the prism and of the table by means of adjusting screws M_1 and M_2.
- *Concerning measurement*: after letting the laser warm up for about 15 minutes, experimental set up is carried out without polarisation filter P_2 to start with.
- To determine the incident intensity I_0'' of the light polarised parallel to the plane of incidence (pointer of polarizer P_1 set to 90), the prism is removed and the rotating guide rail is rotated so that the laser beam falls directly onto the photocell (amplification of the universal measurement amplifier must be adjusted in such a way that voltage does not increase above the maximum output voltage of 10 V).
- After the prism has been replaced onto the prism table, the rotating guide rail with detector LD is set to an angle φ of about 10°. The prism table carrying the prism is now turned so that the reflected beam is directed towards detector LD. According to Snellius's law, the angle of incidence is equal to the exiting angle, that is, angle of incidence α is half the angle φ formed by the incident laser beam and the rotating guide rail.
- The angle of the rotating guide rail is now modified in steps of 5° (steps of about 2.5° in the area of Brewster's angle). The prism is turned every time in such a way that the laser beam falls on the detector, in order to determine light intensity I_r''. Angle φ should be varied up to about 160°.
- This experiment is repeated with light polarised perpendicularly to the plane of incidence of the prism (pointer of polarizer P_1 set to 0°). For this, the intensity of the incident laser beam without prism, $I_0{}^{\perp}$ must be determined to start with.
- Concerning the 2nd part of measurements: polarising filter P_2 is brought into the beam path between prism and photocell on the rotating guide rail. Polarizer P_1 is set to an angle of 45° (pointer set to 45). Without prism, detector LD would indicate an intensity minimum if the polarising directions of the two polarising filters P_1 and P_2 were crossed (P_2 pointer at -45°). One makes use of the fact that the intensity minimum can be determined more precisely than the peak, so that during reflection at the prism, on looks for the intensity minimum through rotation of polarising filter P_2. The rotation supplementary to - 45° is the rotation of the plane of polarisation Ψ due to reflection at the prism. This is carried out for different angles of incidence α of the laser beam on the surface of the prism. Variation of the angle of incidence is carried out as in the first part.

Theory and evaluation

In a light wave, the electric field vector E and the magnetic vector B oscillate perpendicularly and in phase to each other. Intensity is given by Maxwell's equation:

$$|B| = n \cdot |E| \tag{1}$$

where n is the refraction index of the medium through which the light beam travels. The energy of the wave transported in the direction of propagation is given by Poynting's vector, according to the following relation:

$$S \approx E \times B \text{ und } |S| \approx |E|^2 \tag{2}$$

If light falls onto a boundary surface of an isotropic medium of refraction index n with an angle of incidence α, part of the intensity is reflected and the rest goes through the medium under an angle of refraction β. The following indexes are used in the theory mentioned below:

$$x^{\perp}, x^{''}$$

direction of oscillation of the electric and magnetic filed vectors, which are directed either parallel or perpendicularly to the angle of incidence.

$$x_0, x_r, x_t$$

Incident, reflected and refracted vector components. In fig. 2A), the electric field vector

$$E_0{}^{\perp}$$

of the incident light wave oscillates perpendicularly to the plane of incidence; according to (2), magnetic vector B_0'' oscillates parallel to the latter. Related to the law of continuity of the tangential components (that is, the components which oscillate parallel to the surface of the object) and to the direction of the beam, the following relation holds:

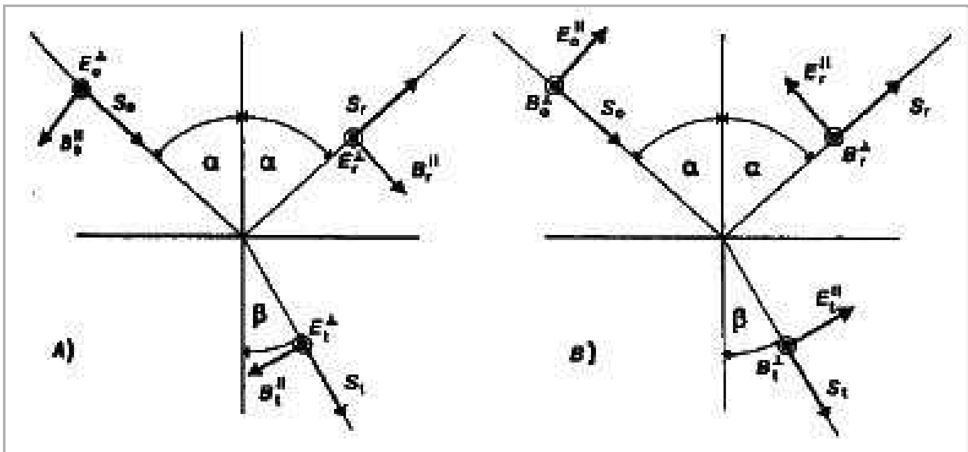

Fig. 2: A) direction of oscillation of the electric field vector perpendicularly and B) parallel to the direction of incidence

$$E_0{}^{\perp} + E_r{}^{\perp} = E_t{}^{\perp} \tag{3}$$
$$(B_0'' - B_r'')\cos\alpha = B_t''\cos\beta$$

With (1) and (3) one obtains:

$$(E_0{}^{\perp} - E_r{}^{\perp})\cos\alpha = n(E_0{}^{\perp} + E_r{}^{\perp})\cos\beta \tag{4}$$

Taking into account the law of refraction, the relation of field intensities is:

$$\zeta^{\perp} = \frac{E_r^{\perp}}{E_0^{\perp}} = \frac{\cos\alpha - n\cos\beta}{\cos\alpha - n\cos\beta} = \frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)} \tag{5}$$

where ζ is defined as reflection coefficient.
Fig. 2B) shows an incident light wave, whose vector E_0'' oscillates perpendicularly to the plane of incidence.
The following is obtained, similar to (3):

$$B_0{}^{\perp} + B_r{}^{\perp} = B_t{}^{\perp} \tag{6}$$

$$E_0'' - E_r'' \cos\alpha = E_t'' \cos\beta$$

$$(E_0{}'' - E_r{}'')\cos\alpha = \frac{1}{n}(E_0{}'' + E_r{}'')\cos\beta \tag{7}$$

$$\zeta'' = \frac{E_r''}{E_0''} = \frac{n\,\cos\,\alpha - \cos\,\beta}{n\,\cos\,\alpha + \cos\,\beta} = \frac{\tan(\alpha-\beta)}{\tan(\alpha+\beta)} \tag{8}$$

Fresnel's formulae (5) and (8) can be written in a different form, eliminating refraction angle β by means of *Snellius' law of refraction*:

$$\zeta^{\perp} = \frac{E_r^{\perp}}{E_0^{\perp}} = -\left(\frac{(\sqrt{n^2 - \sin^2\,\alpha} - \cos\,\alpha)^2}{n^2 - 1}\right) \tag{9a}$$

$$\zeta'' = \frac{E_r''}{E_0''} = \frac{n^2\,\cos\,\alpha - \sqrt{n^2 - \sin^2\,\alpha}}{n^2\,\cos\,\alpha + \sqrt{n^2 - \sin^2\,\alpha}} \tag{9b}$$

$\zeta^{\perp} \geq \zeta''$ is valid for all angles of incidence α between zero and $\pi/2$.

Special cases

A: the following relation is valid for perpendicular incidence ($\alpha = \beta = 0$):

$$\zeta^{\perp} = \zeta'' \left|\frac{n-1}{n+1}\right| \tag{10}$$

B: for a grazing angle $(\alpha = \pi/2$, the following holds:

$$\zeta^{\perp} = \zeta'' = 1 \tag{11}$$

C: if the reflected and the refracted beams are perpendicular to each other $\alpha + \beta = \pi/2$, as shown in fig. 3, there follows from (8):

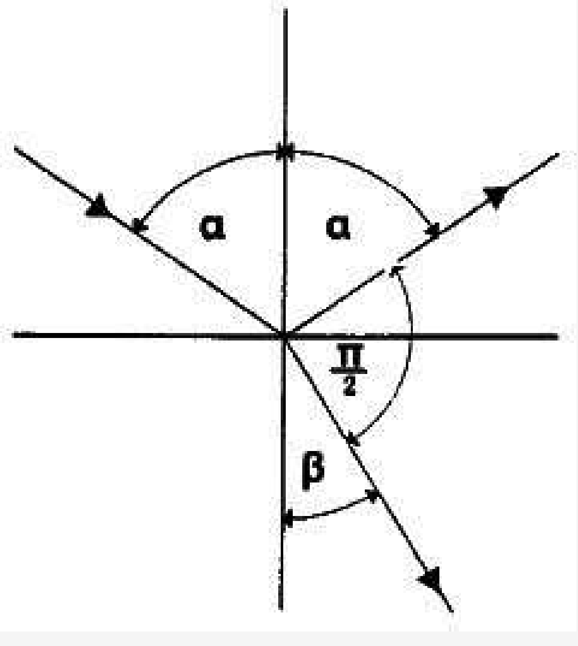

Fig. 3: Brewster's law

$$\zeta'' = 0 \tag{12}$$

that is, reflected light is completely polarised. In this case, the electric vector oscillates only perpendicularly to the plane of incidence. Related to Snellius' law of refraction, the following is valid:

$$\sin\alpha = n \cdot \sin\beta = n \cdot \sin(\tfrac{\pi}{2} - \alpha) = n \cdot \cos\alpha$$

so that for this special case one obtains an angle of incidence from

$$\tan\alpha_p = n \tag{13}$$

(α_p = polarisation or Brewster's angle).

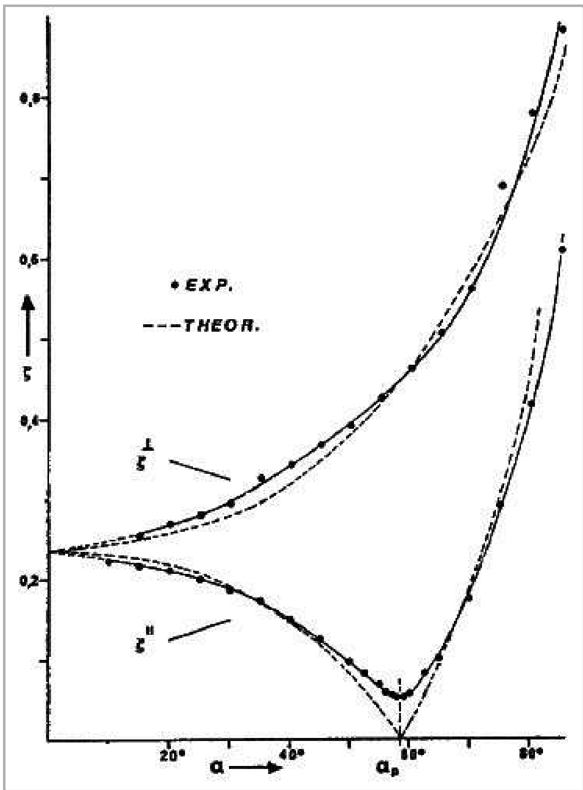

Fig. 4: Measured and calculated curves for ζ_r'' and $\zeta_r{}^{\perp}$ as a function of the angle of incidence

Table 1 contains the voltages U_r measured with the photocell for the reflected light intensity under an angle α. The voltages are directly proportional to light intensity, which itself is proportional to the square of field intensity according to (2).
Fig. 4 shows the curves determined experimentally for $\zeta^{\perp}$ and ζ' as a function of angle of incidence α. The curve for ζ' displays a significant minimum for α_p = 58.5°. With this value and the point of intersection of the ζ curves with the ordinate axis, which can be obtained through extrapolation, applying (13) and (10), one obtains a value of n = 1.63 for the refraction index. The curve calculated theoretically according to (9), with n = 1.63, shows good agreement with experiment. The flatter curve of ζ'' at α_p is caused by the laser light which has a degree of polarisation <1. If the reflection components from (9a) and (9b) are squared and added, one obtains for the reflection factor R for perpendicular incidence:

$$R = \frac{(E_r^{\perp})^2 + (E_r'')^2}{(E_0^{\perp})^2 + (E_0'')^2} = \left(\frac{n-1}{n+1}\right)^2 \tag{14}$$

The reflection factor R for the flint glass prism $(n = 1.63)s$ is thus approximately 0.06.
Another possibility to verify Fresnel's formulae is based on the following method:
Linearly polarised light, with an electric field vector rotated by an azimuth angle δ against the plane of incidence, impinges on a glass reflector. The rotation of the plane of polarisation of the reflected beam is recorded as a function of the angle of incidence. In fig. 5, the plane of the paper represents the reflection surface. If the electric vector oscillates under an angle ω after reflection, the rotation of the plane of polarisation is given by the angle $\Psi = \delta - \omega$. The following relation is true for the field components parallel and perpendicular to the plane of incidence:

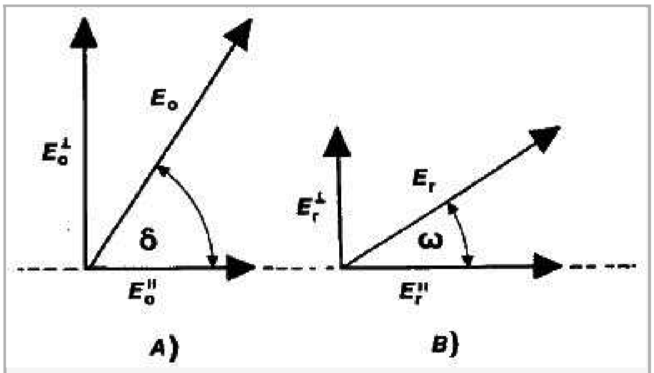

Fig. 5: Rotation of the direction of oscillation through reflection

$$E_r'' = E_r \cos\omega; E_r = E_r{}^{\perp} \sin\omega \tag{15}$$

or

$$\tan\omega = \frac{E_r{}^{\perp}}{E_r''} = \frac{E_r{}^{\perp} . E_0'' . E_0{}^{\perp}}{E_0{}^{\perp} . E_r'' . E_0''} \tag{16}$$

Using (5) and (8), one obtains from (16):

$$\tan\omega = -\left(\frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)}\right) \cdot \left(\frac{\tan(\alpha+\beta)}{\tan(\alpha-\beta)}\right) \cdot \tan\delta \tag{17}$$

For the special case with $\delta = \pi/4$, the following holds:

$$\tan\Psi = \tan\left(\frac{\pi}{4} - \omega\right) = \frac{1-\tan\omega}{1+\tan\omega} \tag{18}$$

If $\tan\omega$ from (17) is inserted into (18), one obtains after transformation:

$$tan\Psi = \frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\cos(\alpha+\beta)-\cos(\alpha-\beta)} = -\left(\frac{\cos\alpha\sqrt{1-\sin^2\beta}}{\sin\alpha \cdot \sin\beta}\right) \tag{19}$$

The definitive formula is obtained through elimination of refraction angle β through the law of refraction:

$$\Psi = \arctan\left(-\left(\frac{\cos\alpha\sqrt{n^2-\sin^2\alpha}}{\sin^2\alpha}\right)\right) \tag{20}$$

If the plane of polarisation is turned by $\Psi = \pi/4$, Brewster’s law results:

$$\tan\alpha_p = n \tag{21}$$

Fig. 6 shows the measured rotation of the plane of polarisation as a function of the angle of incidence with good agreement with the values calculated by means of equation (20).

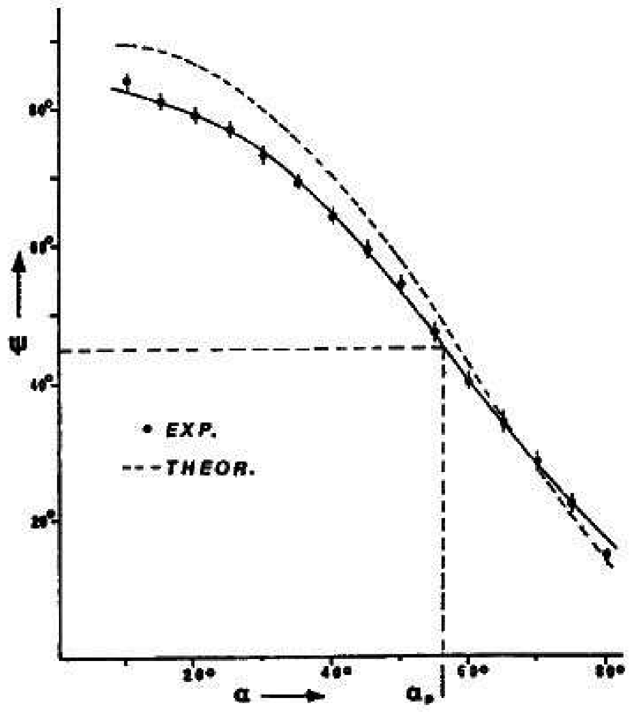

Fig. 6: Measured and calculated curves for the rotation of the direction of oscillation as a function of the angle of incidence (the azimuth of the incident beam is 45°)

α degrees	$U_r{}^{\perp}$ Volt	$\zeta^{\perp}$	U_r'' volt	ζ''
10	0.21	0.264	0.182	0.246
15	0.213	0.266	0.178	0.243
20	0.232	0.278	0.167	0.236
25	0.25	0.288	0.16	0.231
30	0.287	0.309	0.129	0.207
35	0.33	0.392	0.112	0.193
40	0.363	0.348	0.088	0.171
45	0.42	0.378	0.062	0.144
50	0.48	0.4	0.037	0.111
55	0.588	0.443	0.017	0.075
57.5	–	–	0.01	0.058
60	0.71	0.486	~0.000	0.000
62.5	–	–	0.006	0.081
65	0.87	0.542	0.02	0.082
70	1.11	0.608	0.089	0.172
75	1.43	0.69	0.237	0.281
80	1.73	0.77	0.615	0.453

Tabelle 1: Voltages U_r and reflection coefficients as a function of angle $\alpha(U_0 = 3.0V)$.