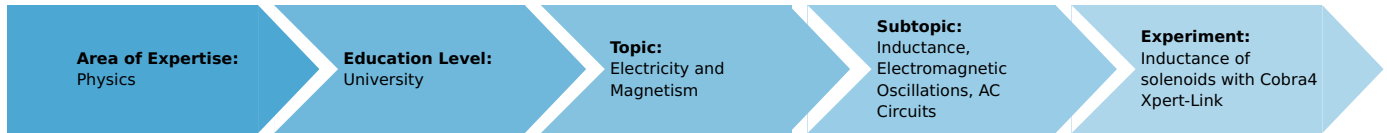


# Inductance of solenoids with Cobra4 Xpert-Link

(Item No.: P2440364)

## Curricular Relevance



### Difficulty



Difficult

### Preparation Time



1 Hour

### Execution Time



2 Hours

### Recommended Group Size



2 Students

### Additional Requirements:

### Experiment Variations:

### Keywords:

Law of induction, Lenz's law, self-inductance, solenoids, transformer, coupled oscillatory circuit, resonance, damped oscillation, logarithmic decrement

## Overview

### Short description

#### Related topics

Law of inductance, Lenz's law, self-inductance, solenoids, transformer, oscillatory circuit, resonance, damped oscillation, logarithmic decrement,  $Q$  factor.

#### Principle

A square wave voltage of low frequency is applied to an oscillating circuit comprising coil and capacitor of known capacitance. The sudden change of voltage at the both edges of the square wave signal induces a magnetic field in the primary coil which then couples into the solenoid and triggers a free damped oscillation in the secondary circuit. For different solenoids the natural frequencies of the circuits are measured and therewith the solenoids' inductances are calculated.

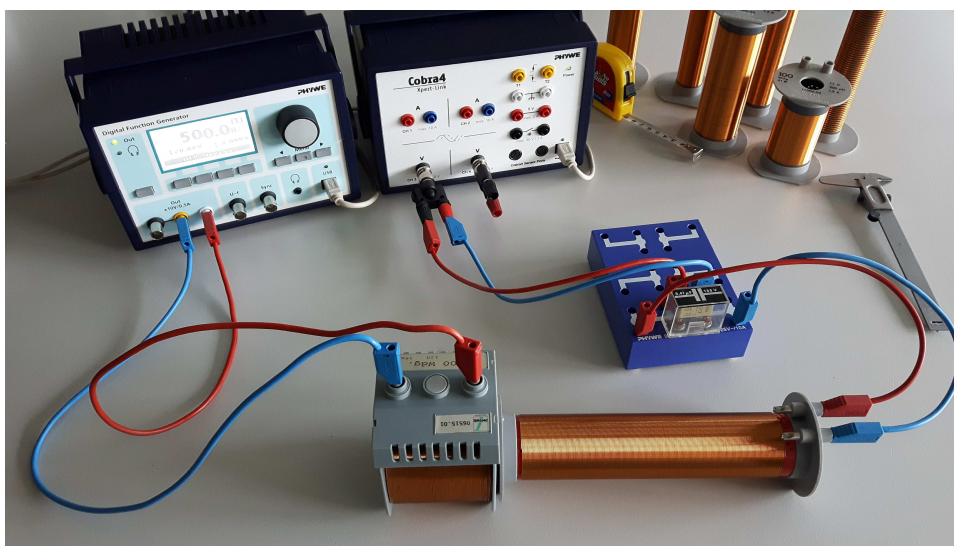


Fig. 1: Experimental setup.

## Equipment

Position No.	Material	Order No.	Quantity
1	Induction coil,300 turns,dia.40mm	11006-01	1
2	Induction coil,300 turns,dia.32mm	11006-02	1
3	Induction coil,300 turns,dia.25mm	11006-03	1
4	Induction coil,200 turns,dia.40mm	11006-04	1
5	Induction coil,100 turns,dia.40mm	11006-05	1
6	Induction coil,150 turns,dia.25mm	11006-06	1
7	Induction coil, 75 turns,dia.25mm	11006-07	1
8	Coil, 1200 turns	06515-01	1
9	Capacitor 470nF/250V, G1	39105-20	1
10	Connection box	06030-23	1
11	Connecting cord, 32 A, 500 mm, red	07361-01	2
12	Connecting cord, 32 A, 500 mm, blue	07361-04	2
13	Connecting cord, 32 A, 500 mm, yellow	07361-02	2
14	Adapter, BNC-plug/socket 4 mm.	07542-26	2
15	PHYWE Digital Function Generator, USB, incl. Cobra4 software	13654-99	1
16	Cobra4 Xpert-Link	12625-99	1
17	Software measureLAB	14580-61	1
Additional material			
	PC, Windows® XP or higher		


## Tasks

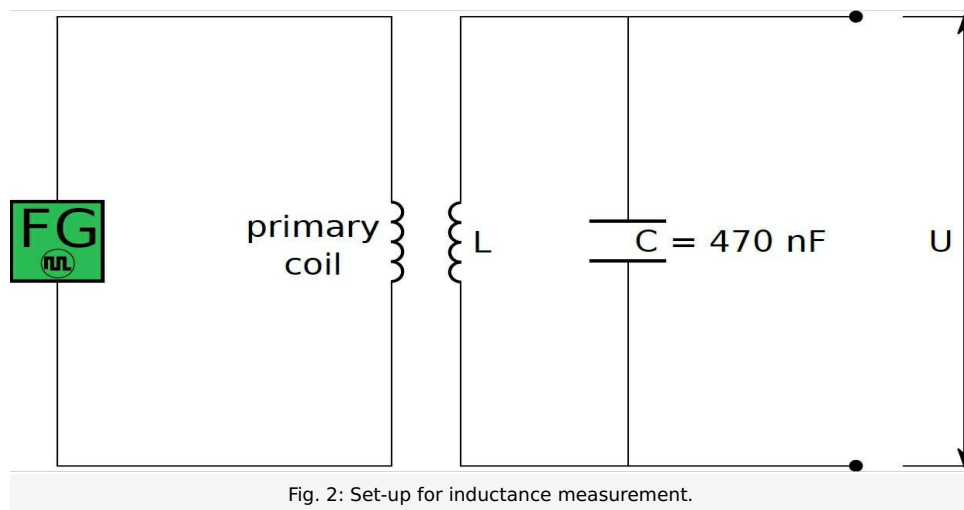
The natural frequency of the induced oscillation has to be measured for each induction coil. From the natural frequency and the known capacitance, calculate the inductances of the coils and determine the relationships between

1. inductance and number of turns,
2. inductance and solenoid length
3. as well as inductance and solenoid radius.



## Set-up and procedure

### Set-up

- Build the electrical circuit according to figs. 1 and 2.
- Start the PC and connect the digital function generator and the Xpert-Link device via USB.
- Start the measureLAB  software package on the PC.
- Choose the appropriate voltage channel of the Xpert-Link device as your measuring channel.
- Load the experiment. All required settings for measuring are set automatically. The digital function generator should have an amplitude of 20 V, frequency of 500 Hz and the square wave signal with signal-type out.
- The solenoid of the LC circuit has to be aligned carefully with the primary coil so that the magnetic field can couple efficiently from the primary coil into the solenoid. The distance between the two coils should be maximized so that the effect of the excitation coil on the resonant frequency can be disregarded. There should be no iron components in the vicinity of the coils.



### Procedure

- Connect the first solenoid to the LC circuit and place it next to the primary coil.
- Start the measurement .
- The measurement stops automatically. The result will be displayed. The time for two damped oscillations should match the period time of the square wave signal (fig. 3).
- Zoom in on one damped oscillation and measure the oscillation period  $T_{\text{exp}}$  of the circuit. Use the measure function  and note your result.

**Note:** In order to reduce measurement errors you can measure the time over several periods of the oscillation and divide your result by the number of periods you have taken.

- Repeat these steps for all the different solenoids. Finally collect all the measured period times in one table.

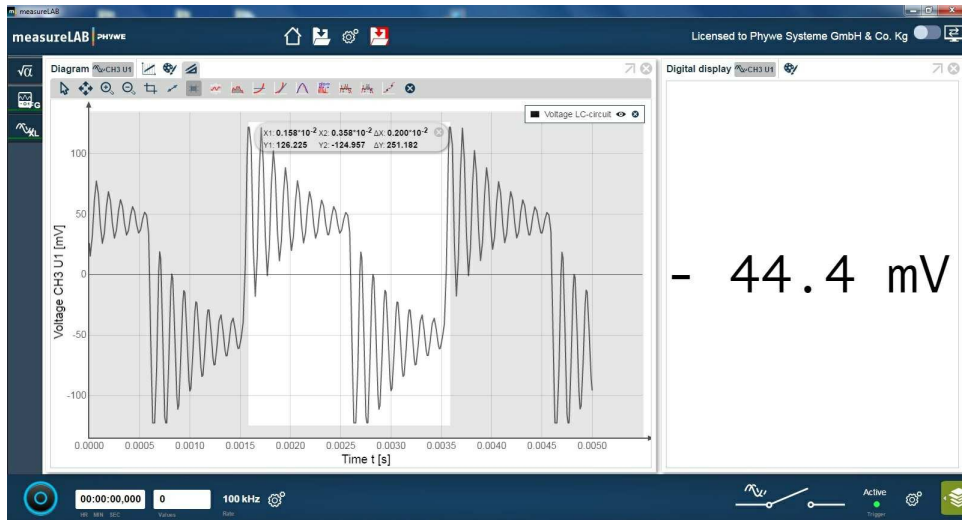


Fig. 3: Damped oscillations. The time for two damped oscillations matches the period time of the square wave signal ( $T=0.002s$ ).

## Theory and evaluation

### Theory

If a current of strength  $I$  flows through a cylindrical coil (a.k.a. solenoid) of length  $l$ , cross sectional area  $A = \pi \cdot r^2$  and number of turns  $N$ , a magnetic field is set up in the coil. When  $l \gg r$  the magnetic field is uniform and the field strength  $H$  is given by

$$H = I \cdot \frac{N}{l} \quad (1).$$

The magnetic flux  $\Phi$  is given by

$$\Phi = \mu_0 \cdot \mu \cdot H \cdot A \quad (2),$$

where  $\mu_0$  is the magnetic field constant and  $\mu$  the absolute permeability of the surrounding medium. When this flux changes it induces a voltage between the ends of the coil

$$U_{\text{ind}} = -N \cdot \frac{d\Phi}{dt} = -N \cdot \mu_0 \mu \cdot A \cdot \frac{dH}{dt} = -L \cdot \frac{dI}{dt} \quad (3),$$

where

$$L = \mu_0 \cdot \mu \cdot \pi \cdot \frac{N^2 \cdot r^2}{l} \quad (4)$$

is the coefficient of self-induction (inductance) of the coil. In practice the inductance of coils with  $l > r$  can be calculated with greater accuracy by an approximation formula

$$L = 2.1 \cdot 10^{-6} \cdot N^2 \cdot r \cdot \left(\frac{r}{l}\right)^{3/4} \quad (5)$$

for  $0 < \frac{r}{l} < 1$ . In the experiment the inductance of various coils is calculated from the period time of an oscillating circuit:

$$\omega_0 = \frac{2\pi}{T_{\text{exp}}} = \frac{1}{\sqrt{LC_{\text{tot}}}}.$$

$C_{\text{tot}} = C + C_i$  is the sum of the known capacitor and the input capacitance  $C_i \approx 40$  pF of the Cobra 4 XpertLink device, which also induces a damping effect on the oscillatory circuit and causes a negligible shift (approx. 1 %) in the resonance frequency. The inductance is therefore represented by

$$L = \frac{T_{\text{exp}}^2}{4\pi^2 C_{\text{tot}}} \quad (6).$$

Tab. 1: Coil data

Coil No.	$N$	$2r$ in mm	$l$ in mm	Cat. No.
1	300	40	160	11006.01
2	300	32	160	11006.02
3	300	26	160	11006.03
4	200	40	105	11006.04
5	100	40	53	11006.05
6	150	26	160	11006.06
7	75	26	160	11006.07

Tab. 2: Measured oscillation periods and corresponding calculated inductances (eq. 6) for LC circuit. For comparison, also theoretically predicted inductances are given (eq. 5).

Coil No.	$T_{\text{exp}}$ in $\mu\text{s}$	$\nu$ in kHz	$L_{\text{exp}}$ in $\mu\text{H}$	$L_{\text{theo}}$ in $\mu\text{H}$
1	119,71	8,35	772,26	830
2	98,00	10,20	517,56	568
3	77,29	12,94	321,92	374
4	95,57	10,46	492,21	506
5	63,00	15,87	213,89	221
6	39,29	25,45	83,19	93
7	20,00	50	21,56	23

### Evaluation








**Note:** To do the evaluation, you need to fit the experimental data to functions of a given form. You can choose any software you like to do this step. For this manual, gnuplot has been used.

The following coils provide the relationships between inductance and 1. number of turns  $N$ , 2. length  $l$  and 3. radius  $r$  that we are investigating:

- no. 3, 6, 7
- no. 1, 4, 5
- no. 1, 2, 3

As a difference in length also means a difference in the number of turns, the relationship between inductance and number of turns found in task 1 must also be used to solve task 2.

In table 2 the measured period lengths for the oscillations in the LC circuits are shown for all the different solenoids used. From these values, the experimentally obtained inductances  $L_{\text{exp}}$  can be obtained from equation 6. The theoretically predicted values  $L_{\text{theo}}$  from equation 5 are shown as well. The experimental values  $L_{\text{exp}}$  are in good agreement with the theoretically expected values  $L_{\text{theo}}$ .

You can plot these values with measureLAB. Go to  and generate two data sets with your obtained experimental values and the corresponding theoretically calculated values with . After generating the data sets, select both in the data pool  and choose the option  to display a diagram. In the diagram, go to  and select the data set for the theoretical values as x-axis. Go to  and select 'Points' to display a scatter plot of your data (c.f. Fig. 4). Go back to  and select to generate a regression line.

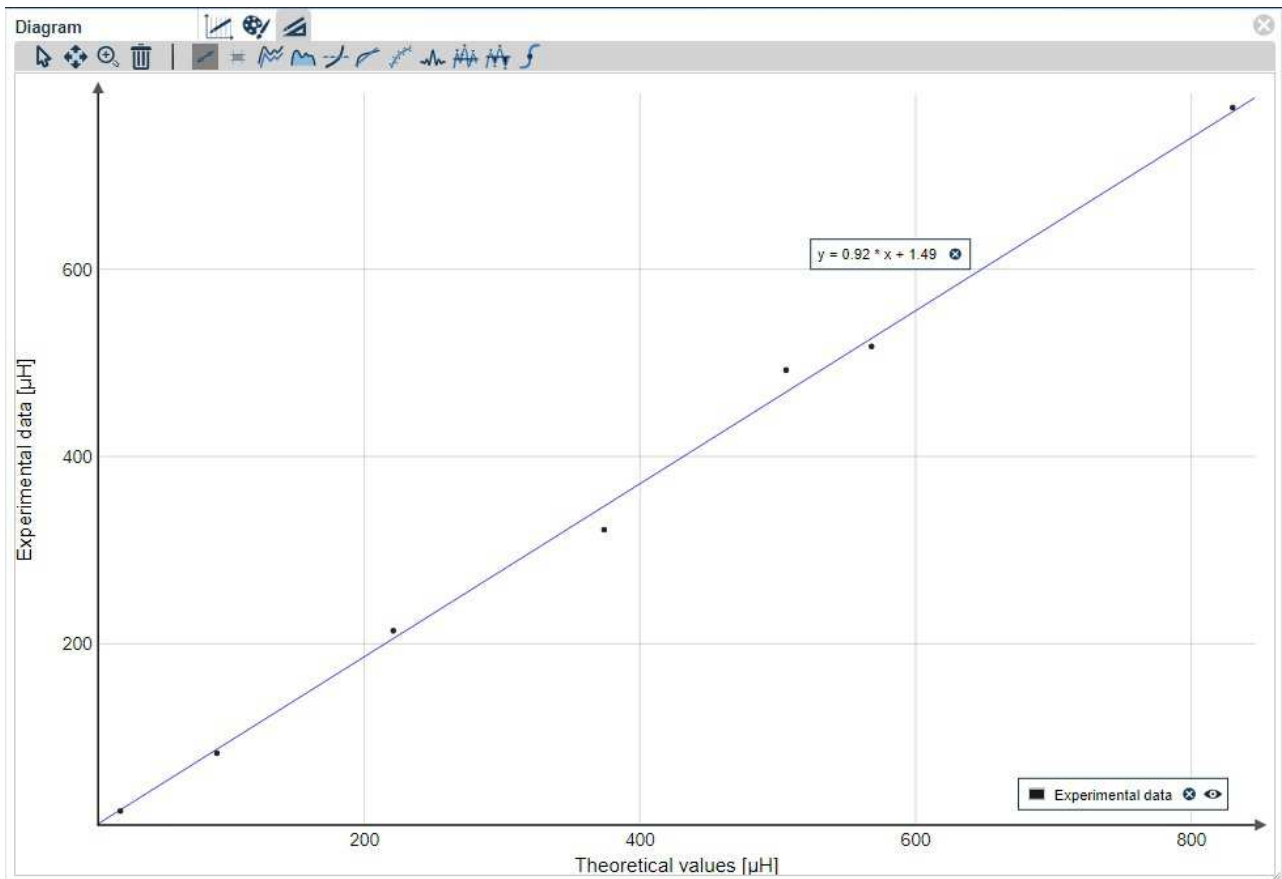









Fig. 4: Comparison of experimentally obtained values for the inductances of the LC circuit solenoids with theoretically calculated values in a scatter diagram.

### 1. Task: Determine the coils' relationship between inductance and number of turns.

- To determine the relationship between inductance and number of turns consider coils with identical radius and length but different number of turns. Coils no. 3, 6 and 7 meet these requirements.

As performed in Task 1, go to  and generate the respective data sets (Inductance L for coils no. 3,6 and 7; corresponding Number of turns N) with . Subsequently, select both data sets to be displayed in a diagram . Following this, go to  and select Number of turns as x-axis. To display the data set as points, go to  and change the settings to points. Lastly, go to  and select  to perform a curve fitting.

Fitting the expression  $L_{\text{exp}} = a_n \cdot x^b$  to the experimental values yields

$$b = 1.951 \pm 0.004$$

which is in excellent agreement with the theoretical value  $b_{\text{theo}} = 2$  (eq. 5).

In Figure 5 the corresponding inductances are plotted in dependence of the number of turns.

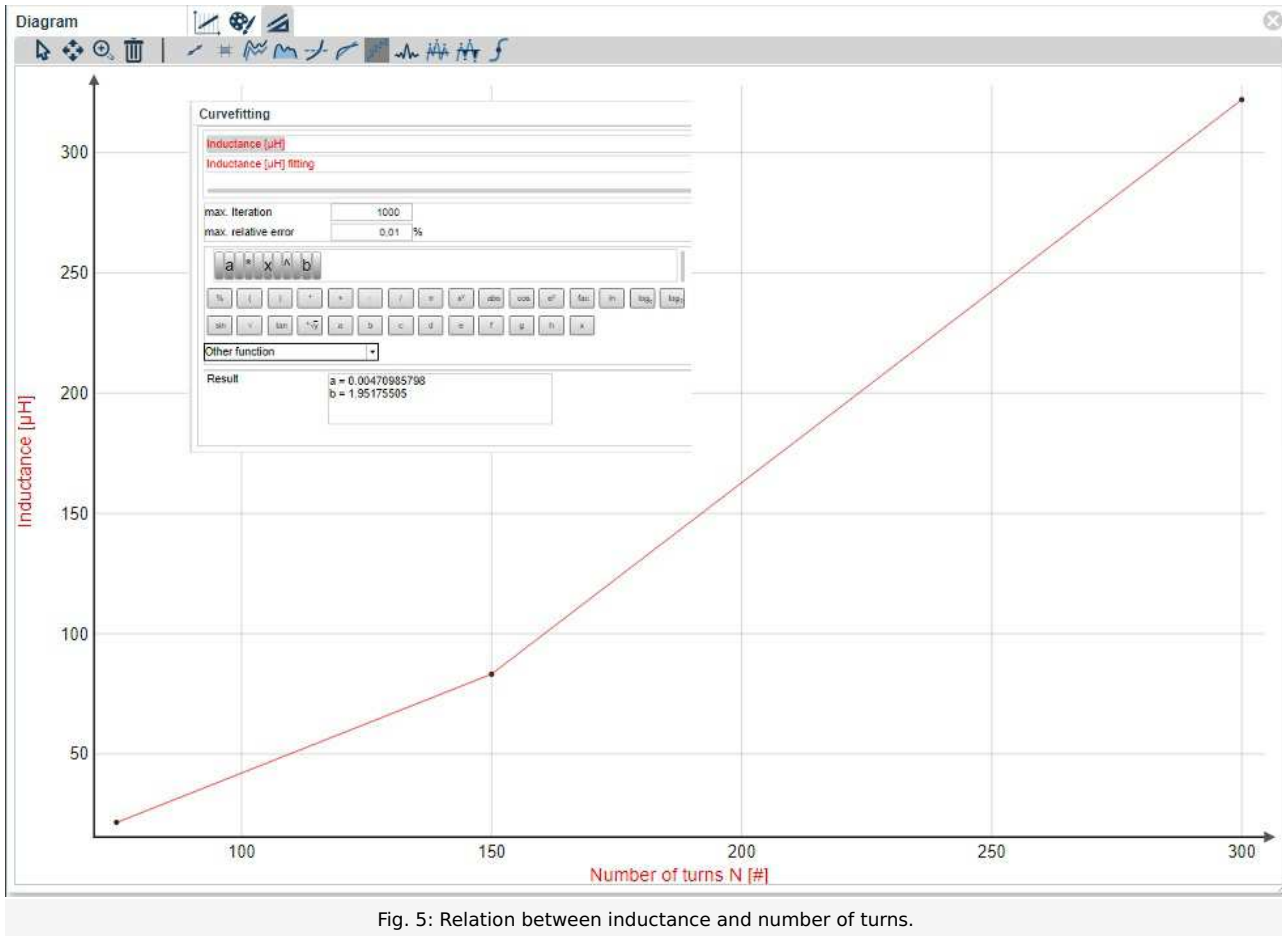


Fig. 5: Relation between inductance and number of turns.

**2. Task: Determine the coils' relationship between inductance and length of coil.**

- To determine the relationship between inductance and length of coil consider coils with identical radius but different lengths. The Coils no. 1, 4 and 5 meet these requirements. As the relation between inductance and number of turns is already known, the inductances can be normalized by the number of turns. Therefore consider the relationship between inductance normalized by turn number squared and the length of coil.

To calculate  $N^2$ , go to and select  $\sqrt{x}$ . Drag and drop your data set of the respective Number of turn N values and calculate the squared values. In the next step, normalise the experimental inductance values to the number of turns; go to and select  $\sqrt{x}$ . Drag and drop the respective data sets to obtain the values for  $L_{exp}/N^2$ . Plot the data as described in Task 1.

Fitting the expression  $L_{exp}/N^2 = a_l \cdot x^c$  to the experimental values yields

$$c = -0.81 \pm 84$$

which is in fair agreement with the theoretical value  $c_{theo} = -0.75$  (eq. 5).

In Figure 6 the corresponding values are plotted in dependence of the coil length.

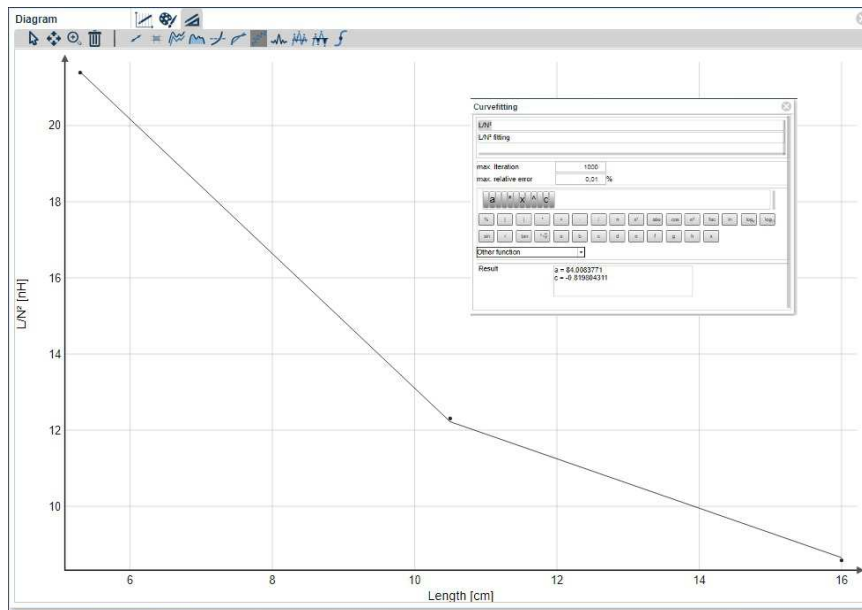


Fig. 6: Relation between inductance and length of coil.

**3. Task: Determine the coils' relationships between inductance and radius of the coils.**

- To determine the relationship between inductance and radius of coil consider coils with identical lengths but different radii. The Coils no. 1, 2 and 3 meet these requirements.

For data analysis, proceed as described in Task 1 ff.

Fitting the expression  $L_{\text{exp}} = a_r \cdot x^d$  to the experimental values yields

$$d = 1.95 \pm 2.19$$

which is in good agreement with the theoretical value  $d_{\text{theo}} = 1.75$  (eq. 5).

In Figure 7 the corresponding values are plotted in dependence of the coil radius.



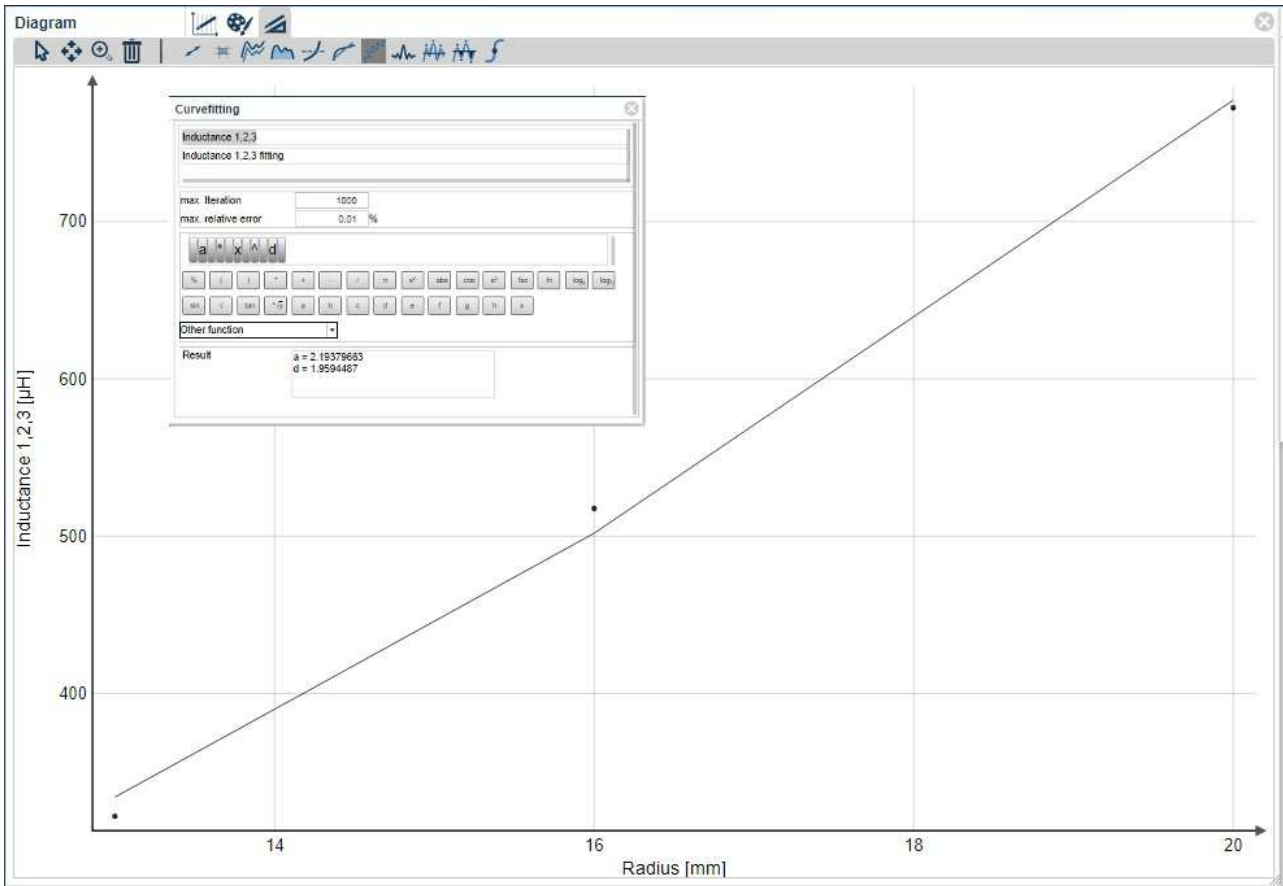


Fig. 7: Relation between inductance and radius of the coil.