Velocity of molecules and the Maxwell-Boltzmann distribution function (Item No.: P2320300)
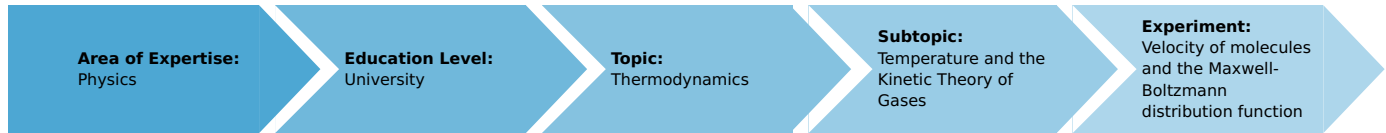
Curricular Relevance

Area of Expertise: Physics	Education Level: University	Topic: Thermodynamics	Subtopic: Temperature and the Kinetic Theory of Gases	Experiment: Velocity of molecules and the Maxwell-Boltzmann distribution function

Difficulty	Preparation Time	Execution Time	Recommended Group Size
Intermediate	10 Minutes	10 Minutes	2 Students

Additional Requirements:

- Precision balance, 620 g / 0.001 g

Experiment Variations:

Keywords:

Kinetic theory of gases, Model kinetic energy, Average velocity, Velocity distribution

Overview

Short description

Principle

A kinetic gas theory apparatus is to be used as a model to simulate the motion of gas molecules and to determine their velocity by determining the distance that glass balls are thrown. The velocity distribution found is to be compared with the theoretical Maxwell-Boltzmann equation.

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Fig. 1: Experimental setup.

Safety instructions

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Equipment

Position No.	Material	Order No.	Quantity
1	Kinetic gas theory apparatus	09060-00	1
2	Receiver with recording chamber	09061-00	1
3	PHYWE power supply, variable DC: 12 V, 5 A / AC: 15 V, 5 A	13540-93	1
4	Tripod base PHYWE	02002-55	2
5	Digital stroboscope	21809-93	1
6	Stopwatch, digital, 1/100 s	03071-01	1
7	Beaker, high, BORO 3.3, 50 ml	46025-00	5
8	Spoon,stainless steel,210mm	40874-00	1
9	Connecting cord, 32 A, 750 mm, red	07362-01	1
10	Connecting cord, 32 A, 750 mm, blue	07362-04	1

Tasks

1. Measure the velocity distribution of the "model gas".
2. Compare the result to theoretical behaviour as described by the MAXWELL-BOLTZMANN distribution.
3. Discuss the results.

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Setup and Procedure
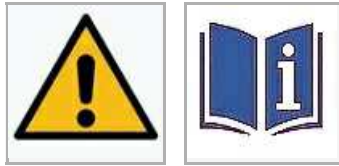
Set up the experiment as shown in Fig. 1.

Fit the receiver with recording chamber to the apparatus as described in the operating instructions for the determination of particle velocities.

First determine the average weight of one glass ball by weighing out a known number of balls (e.g. 100) to avoid the time consuming counting of glass balls during the experiment.

Following this, determine the average number of glass balls expelled from the apparatus during 1 minute. To do this, fill the apparatus with 400 glass balls and set it to the following conditions:

- height of the upper piston: 6 cm
- oscillator frequency: 50 s^{-1} (controlled by the voltage and the stroboscope).

Now open the outlet for 1 minute and determine the number of balls expelled by weighing them. Refill the apparatus with these balls and repeat the experiment twice.

Prepare for the simulation experiment by calculating the average number of balls expelled per minute and filling this number of glass balls into each of four glass beakers. Make the following apparatus set-tings:

- height of the upper piston: 6 cm
- height difference between outlet and receiver: 8 cm
- number of balls: 400
- oscillator frequency: 50 s^{-1}.

When the frequency is stable, open the outlet for 5 minutes.

After each minute, pour the balls from one of the beakers into the apparatus to maintain a constant 'par-ticle density'. Determine the number of glass balls in each of the 24 compartments of the receiver by weighing.

Refill the glass beakers and repeat the experiment four times.

Theory and evaluation

According to definition, the kinetic energy of the molecules of an ideal gas is given by

$$\bar{E}_k = \frac{m}{2} \cdot c^{-2} \text{ (1)}$$

E_k Average kinetic energy

m Mass of the molecule

c Average velocity of the molecule

From kinetic theory, the pressure of an ideal gas can be described by

$$p = \tfrac{1}{3} \cdot \rho \cdot c^2 \text{ (2)}$$

p Pressure

r Density

Combining equations (1) and (2) together with the ideal gas law

$$p \cdot V_{mol} = R \cdot T \text{ (3)}$$

V_{mol} Molar volume

R Gas constant

T Absolute temperature

leads to the following expression for c:

$$\bar{c} = \left(\frac{3 \cdot R \cdot T}{M}\right)^{1/2}$$

or

$$\bar{c} = \left(\frac{3 \cdot k \cdot T}{m}\right)^{1/2} \text{ (4)}$$

k Boltzmann constant

This means that the average kinetic energy is directly proportional to the absolute temperature of the gas which is the interpretation of temperature on the molecular level.

The direct determination of the velocity of a certain molecule is impossible because collisions with other molecules cause it to change incessantly. For a great number of molecules, a distribution function for molecular velocities can be derived by means of statistical methods. This was done by Maxwell and Boltzmann with the following result:

$$\frac{dN}{N} = \sqrt{\frac{2}{\pi}} \cdot \left(\frac{m}{k \cdot T}\right)^{3/2} \cdot c^2 \cdot e^{-\left(\frac{m \cdot c^2}{2kT}\right)} \cdot dc \text{ (5)}$$

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This equation describes the probability that the velocity of a molecule is within the interval {c,c+dc}. The corresponding distribution function for oxygen at 273 K is shown in Fig. 2 as an example.
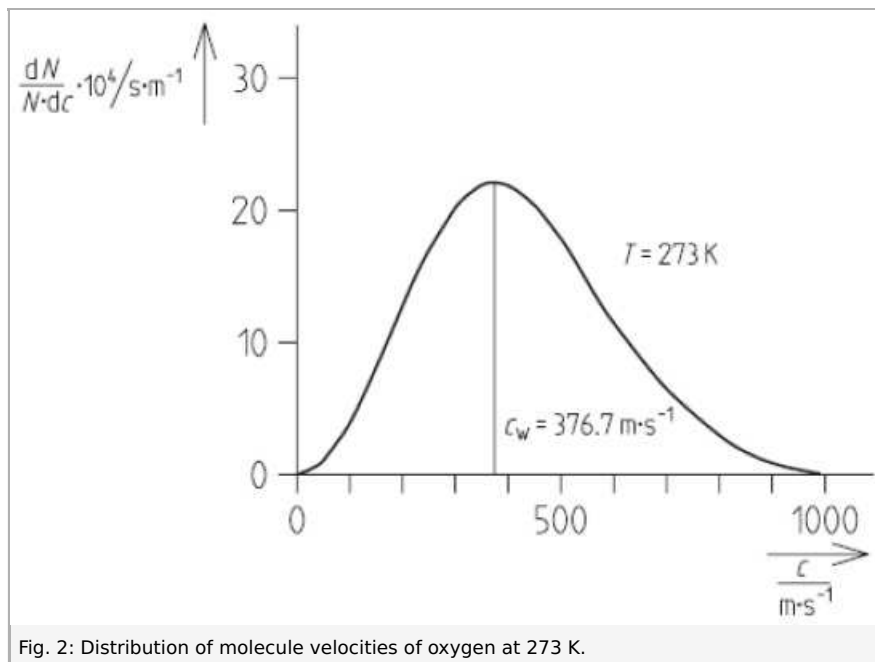

Fig. 2: Distribution of molecule velocities of oxygen at 273 K.

For the velocity at the maximum cw of the curve (velocity with highest probability) the following relation can be derived:

$$c_w = \left(\frac{2 \cdot k \cdot T}{m}\right)^{1/2} \quad (6)$$

Introducing of equation (6) into equation (5) leads to

$$\frac{dN}{N} = \frac{4}{\sqrt{\pi}} \cdot \left(\frac{1}{c_w^2}\right)^{3/2} \cdot c^2 \cdot e^{-\frac{c^2}{c_w^2}} \quad (7)$$

Note that $c_w \neq c$ and

$$c_w : \bar{c} : \sqrt{\overline{c^{-2}}} = \sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3} = 1 : 1.13 : 1.22$$

In the model experiment with glass balls, the velocity of the balls can be determined from the distance thrown s :

$$c = s \cdot \left(\frac{g}{2 \cdot h}\right) = K \cdot s \quad (8)$$

g Acceleration at the earth surface (= 9.81 ms)

h Height difference between outlet and receiver

Now the experimental results (number of balls per distance thrown interval) can be displayed graphically in the form

$$\frac{1}{\sum N_i} \cdot \left(\frac{N_i}{\Delta c}\right) = f(c) \quad \text{(9)}$$

N_i Number of balls in the interval i, i = 1...23

Δc Velocity interval corresponding to Δs = 1 cm (0.078 ms)

as shown in Figure 3.

Fig. 3: Experimental and theoretical velocity distribution in the model experiment.
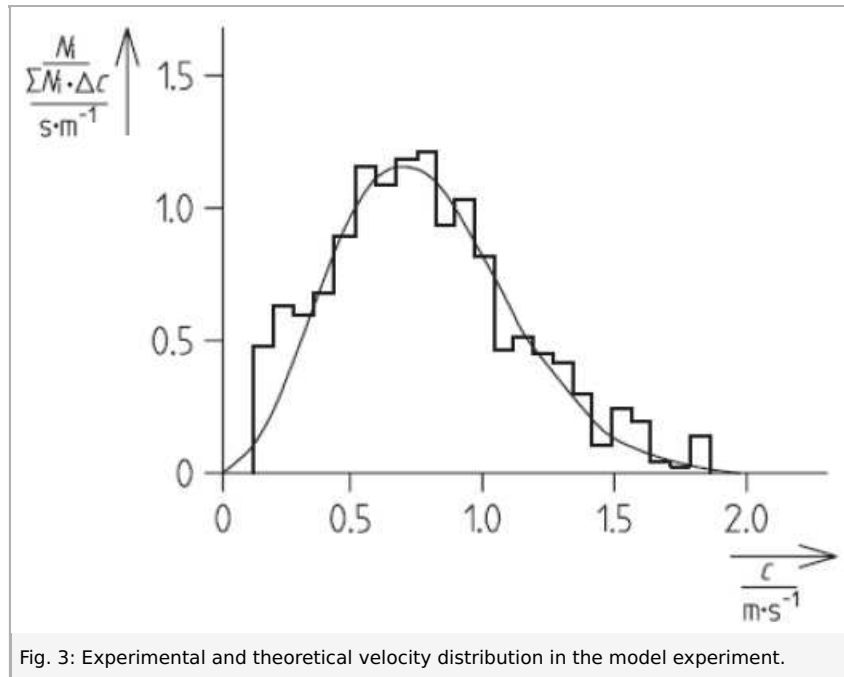
The theoretical distribution function can be evaluated by means of equation (7) using the velocity at the maximum of the experimental distribution as c_w . The result for the example in Fig. 3 is also shown in the diagram. The agreement between the two curves is reasonably good taking into account the model cha-racter of the experiment.

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