Mechanical energy conservation/Maxwell's wheel with measure Dynamics (Item No.: P2131880)

Curricular Relevance

Area of Expertise: Physics
Education Level: University
Topic: Mechanics
Subtopic: Rotational Motion
Experiment: Mechanical energy conservation/Maxwell's wheel with measure Dynamics

Difficulty	Preparation Time	Execution Time	Recommended Group Size
Difficult	10 Minutes	20 Minutes	2 Students

Additional Requirements:

Experiment Variations:

Keywords:

Maxwell's wheel, kinetic energy, rotational energy, potential energy, moment of inertia, angular velocity, angular acceleration, instantaneous velocity, gyroscope

Introduction

Overview

A wheel, which can unroll around its axis on two cords, moves a gravitational field. This process is filmed with a video camera. The potential energy, kinetic energy, and rotational energy are converted into one another and determined as a function of time with the aid of the "measure Dynamics" software.

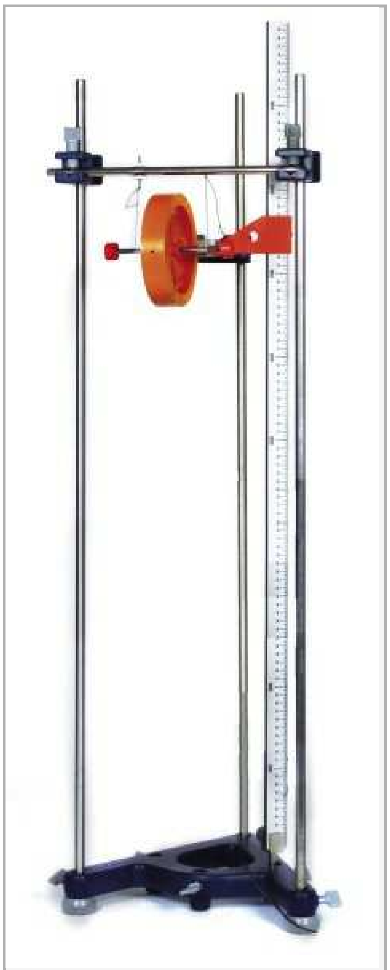

Fig. 1: Experiment set-up

Equipment

Position No.	Material	Order No.	Quantity
1	Support base DEMO	02007-55	1
2	Support rod, stainless steel, 1000 mm	02034-00	3
3	Right angle clamp expert	02054-00	3
4	Meter scale, l = 1000 mm	03001-00	1
5	Maxwell wheel	02425-00	1
6	Holding device with cable releasee	02417-04	1
7	Plate holder	02062-00	1
8	Software "Measure Dynamics", single user	14440-61	1

Additional equipment:

Video camera, tripod, computer

Tasks

1. Determination of the moment of inertia of the Maxwell wheel by way of the distance-time relation-ship.
2. Determination of the moment of inertia of the Maxwell wheel by way of the velocity-time relation-ship.
3. Graphical representation of the potential energy, kinetic energy, and rotational energy as a function of time.

Set-up and procedure

Set the experiment up as shown in Figure 1. Use the adjustable screw on the support rod for the horizontal alignment of the axis of the Maxwell wheel in the unwound state. Then, wind the Maxwell wheel uniformly up on both sides. When doing so, ensure that the windings run inwards.
It is essential to observe the first up and down movement of the wheel, since incorrect winding (outwards, crossed) may cause the Maxwell wheel to break free.
The release switch, i.e. the pin that is located in a hole of the Maxwell wheel, is used for releasing the wheel. Adjust the release switch so that the wheel neither oscillates nor rolls after it has been released. In addition, ensure that the cords are always wound in the same direction for the experiments.
In terms of the video that will be recorded, the following must be taken into consideration concerning the setting and positioning of the camera:

- Set the number of frames per second to approximately 30 fps.
- Select a light-coloured, homogeneous background.
- Provide additional lighting for the experiment.
- The experiment set-up should be in the centre of the video. To ensure this, position the video camera on a tripod centrally in front of the experiment set-up.
- Film the experiment from the side.
- The experiment set-up should fill the video image as completely as possible.
- The optical axis of the camera must be parallel to the experiment set-up (no movement in the x-direction).
- For scaling, position a scale next to the experiment set-up by way of a support base, support rod, right-angle clamp, and plate holder.

Then, the video recording process and the experiment can be started.

Theory and evaluation

The total energy E of the Maxwell wheel with the mass m and the moment of inertia I_z around its axis of rotation is composed of the potential energy E_{pot}, kinetic energy E_{kin}, and rotational energy E_{rot} as follows:

$$E = m \cdot \vec{g} \cdot \vec{s} + \frac{m}{2}\vec{v}^2 + \frac{I_z}{2}\vec{\omega}^2$$

Here,
$\vec{\omega}$ is the angular velocity,
$\vec{v}$ is the translational velocity (velocity of the centre of gravity of the wheel in the direction of movement),
$\vec{g}$ is the gravitational acceleration, and
$\vec{s}$ is the (negative) height.

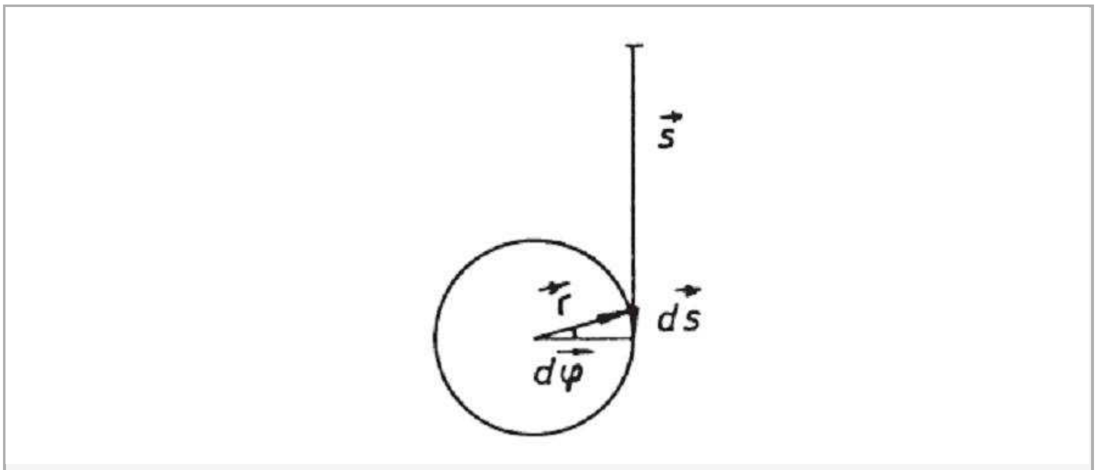

Figure 2: Relationship between the increase in dφ and the decrease in height ds .

With the notation of Figure 2, the relationships

$$d\vec{s} = d\vec{\varphi} \times \vec{r}$$

and

$$\vec{v} = \frac{d\vec{s}}{dt} = \frac{d\vec{\varphi}}{dt} \times \vec{r} = \vec{\omega} \times \vec{r}$$

result, where $\vec{r}$ is the radius of the wheel.
In the present case, $\vec{g}$ is parallel to $\vec{s}$ and $\vec{\omega}$ is perpendicular to $\vec{r}$, so that

$$E = -m \cdot g \cdot s(t) + \frac{1}{2} \cdot (m + \frac{I_z}{r^2}) \cdot (v(t))^2$$

applies. Since the total energy E is constant, differentiation leads to

$$\frac{dE}{dt} = 0 = -m \cdot g \cdot v(t) + (m + \frac{I_z}{r^2}) \cdot v(t) \cdot \dot{v(t)}$$

For $s(t=0) = 0$ and $v(t=0) = 0$ we obtain

$$s(t) = \frac{1}{2} \cdot \frac{m \cdot g}{m + \frac{I_z}{r^2}} \cdot t^2 \quad (1)$$

and

$$v(t) = \frac{ds}{dt} = \frac{m \cdot g}{m + \frac{I_z}{r^2}} \cdot t \quad (2)$$

The second experiment evaluation requires the following data: The mass m of the wheel during the experiment is m = 0.436 kg. The radius of the wheel is r = 2.5 mm.
Transfer the video that has been recorded to the computer. Then, start "measure Dynamics" and open the video under "File" – "Open video ...". Mark the start of the experiment ("Start selection" and "Time zero") and the end of the experiment ("End selection") in the video for further analysis via the menu line above the video. The experiment begins when the Maxwell wheel starts to unroll and it ends when the first turning point is reached. Then, mark the scale with the scale that appears in the video by way of "Video analysis" – "Scaling ..." – "Calibration" and enter the resulting length into the input window. In addition, enter the frame rate that has been set for the recording process under "Change frame rate" and position the origin of the system of coordinates at the centre of the Maxwell wheel at the beginning of the experiment under "Origin and direction".
Then, the actual motion analysis can be started under "Video analysis" – "Automatic analysis" or "Manual analysis". For the automatic analysis, we recommend selecting "Motion and colour analysis" on the "Analysis" tab. Under "Options", the automatic analysis can be optimised, if necessary, e.g. by changing the sensitivity or by limiting the detection radius. Then, look for a film position in the video where the centre of the Maxwell wheel is perfectly visible. Click the centre. If the system recognises the object, a green rectangle appears and the analysis can be started by clicking "Start".
If the automatic analysis does not lead to any satisfying results, the series of measurements can be corrected under "Manual analysis" by manually marking the centre of the Maxwell wheel.

Task 1: Determination of the moment of inertia of the Maxwell wheel by way of the distance-time relationship.

Add a new column to the worksheet by clicking "New column" in the table menu line. Enter the distance "s" that has been covered into the new column (unit: "m"; formula: "-y"). In order to display the curve of the distance covered as a function of time, select "Display" and "Diagram", click "Options", delete all of the already existing graphs, and select the graph t (horizontal axis) – s (vertical axis). This leads to:

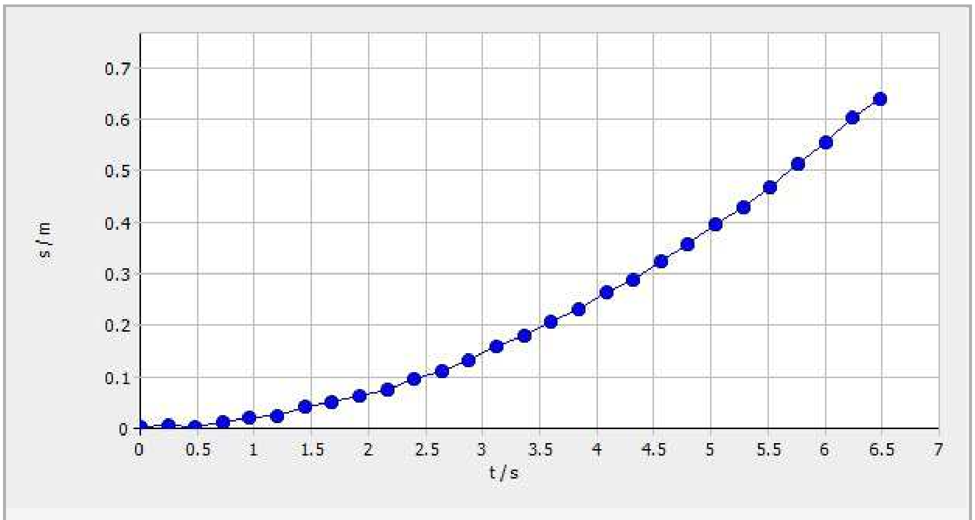

Figure 3: Distance covered s as a function of time t

As it could have been expected from equation (1), Figure 3 shows a quadratic relationship. In order to examine it in greater detail, it is advisable to consider the dependence of the distance covered on the square of the time. To be able to visualise this is graphical form, the worksheet must be extended by clicking "New column" in the table menu line in order to add a new column. Then, enter the square of the time "t2" (unit: "s^2"; formula: "t^2") into the new column. As a result, the t^2-s diagram can be displayed in the same way as the t-s diagram. The following results:

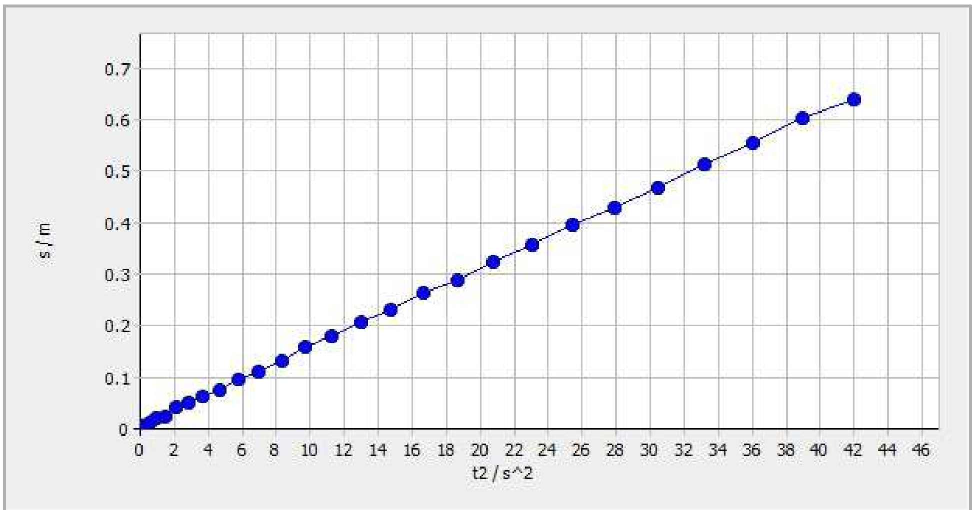

Figure 4: Distance covered s as a function of the square of the time t

As shown in Figure 4, the distance covered is linear with regard to the square of the time. Clicking "Options" in the menu line of the diagram and selecting the tab "Linear regression" will add a regression line to the diagram so that the regression line of the distance s as a function of the square of the time t2 can be determined. The result is a gradient of 0.0152.
(1) leads to a moment of inertia of
$I_z = 8.77 \cdot 10^{-4}$ kgm²

Task 2: Determination of the moment of inertia of the Maxwell wheel by way of the velocity-time relation-ship.

It is also possible to determine the moment of inertia via the representation of the velocity of the centre of gravity of the Maxwell wheel as a function of time. Add another new column to the worksheet by clicking "New column" in the table menu line. Enter the velocity "v" (unit: "m/s"; formula: "-v_y") into the new column. In order to display the curve of the velocity of the centre of gravity of the Maxwell wheel as a function of time, select "Display" and "Diagram", click "Options", delete all of the already existing graphs, and select the graph t (horizontal axis) – v (vertical axis).

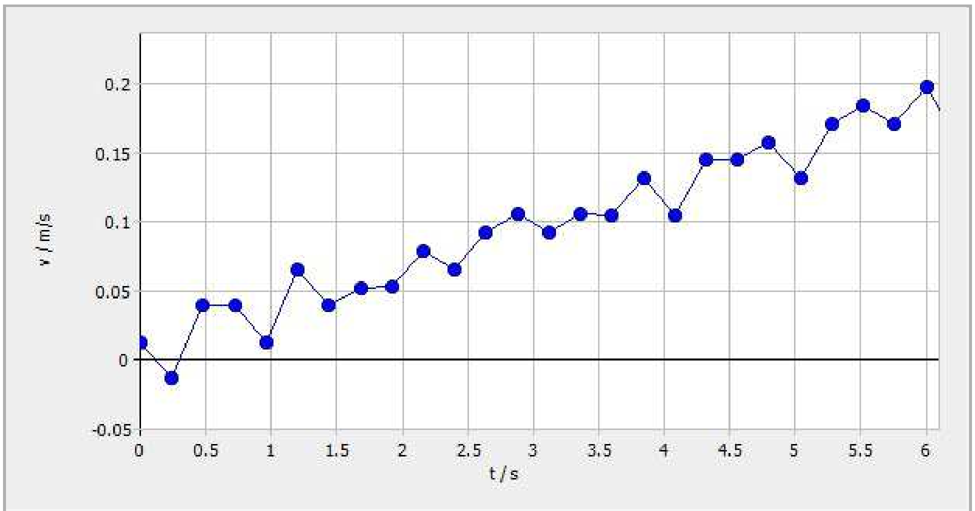

Figure 5: Velocity v as a function of time t

As it could have been expected from equation (2), Figure 5 shows a linear relationship. Clicking "Options" in the menu line of the diagram and selecting the tab "Linear regression" will add a regression line to the diagram so that the regression line of the velocity v as a function of time t can be determined. The result is a gradient of 0.0303.
(2) leads to a moment of inertia of
$I_z = 8.80 \cdot 10^{-4}$ kgm²
Apart from the rounding errors, the two moments of inertia, which have determined in two different ways, are identical.

Task 3: Graphical representation of the potential energy, kinetic energy, and rotational energy as a function of time.

In order to visualise the (negative) potential energy (name: "-E_pot"; unit: "J"; formula: "0.436*9.81*s"), kinetic energy (name: "E_kin"; unit: "J"; formula: "0.5*0.436*(v_y)^2"), and rotational energy (name: "E_rot"; unit: "J"; formula: "0.5*8.77*10^(-4)* (v_y)^2/(2.5*10^(-3))^2") graphically, another new column must be added to the worksheet for each of these types of energy. The graphical representation of the energy types as a function of time leads to:

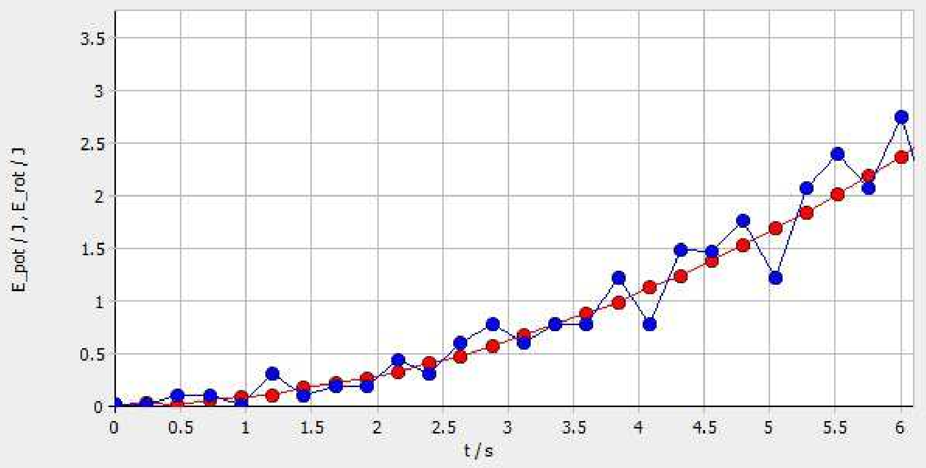

Figure 6: The potential energy (red) and rotational energy (blue) as a function of time t

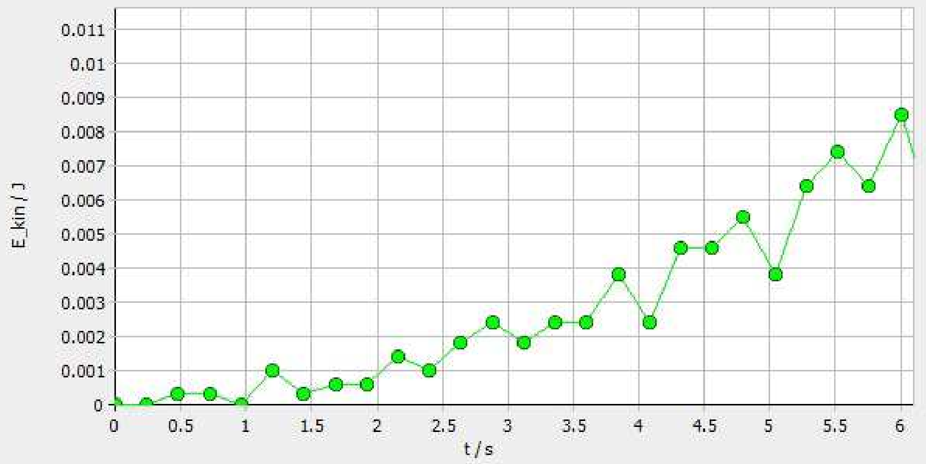

Figure 7: Kinetic energy as a function of time t

Figures 6 and 7 show that the potential energy is nearly completely converted into rotational energy.