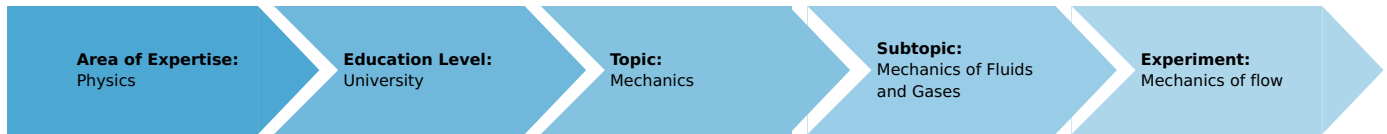


# Mechanics of flow (Item No.: P5140100)

## Curricular Relevance



**Difficulty**



Difficult

**Preparation Time**



20 Minutes

**Execution Time**



2 Hours

**Recommended Group Size**



2 Students

**Additional Requirements:**

- PC

**Experiment Variations:**

**Keywords:**

Ultrasonic Doppler effect, steady flow, laminar and turbulent flow, continuity equation, Bernoulli's equation, Hagen-Poiseuille law, flow velocity, flow resistance, static and dynamic pressure, pressure scales, viscosity and fluidity, and ultrasonic flow measurements

## Overview

### Principle

The ultrasonic Doppler effect is used for studying the laws of steadily and laminarly flowing liquids in a tube circuit, which form the basis of numerous technical applications. The experiment focuses particularly on the relationship between the flow velocity and the crosssectional tube area (continuity condition) as well as on the relationship between the flow resistance and the tube diameter (Hagen-Poiseuille law). If the geometry is known, both relationships can be used to determine the dynamic viscosity or fluidity.



Fig 1: Examination of flowing liquids with Doppler ultrasound techniques, experiment set-up

## Equipment

Position No.	Material	Order No.	Quantity
1	Basic set Ultrasonic Doppler technique	13923-99	1
2	Extension Set: Mechanics of Flow	13923-01	1

Basic set: Doppler ultrasound techniques comprising:

- 1 x Doppler ultrasound unit
- 1 x centrifugal pump
- 1 x ultrasound gel
- 1 x sonography liquid, 1 l
- 1 x ultrasonic probe, 2 MHz
- 1 x Doppler prism, 3/8
- 1 x set of tubes
- 1 x software "measure ultraflow"

Extension set: Laws of flow comprising:

- 1x prism set with flexible and rigid tubes
- 1x manometer tubes (4) on a board with a stand

### Additional equipment

PC (Windows)

## Tasks

1. Determine the mean and maximum Doppler frequency shift ( $f_{\text{mean}}$ ,  $f_{\text{max}}$ ) for four different pump speeds and three different tube diameters with the Doppler ultrasound unit.
2. Calculate the mean flow velocity in accordance with the Doppler law as well as the flow rates based on the known tube cross-section and in accordance with the continuity equation.
3. Measure the pressure drop for the various different flow velocities and tube diameters with the aid of the vertical tubes.
4. Calculate the flow resistance in accordance with Ohm's law based on the pressure drop and on the flow rate. Examine the dependence of the flow resistance on the tube diameter (Hagen-Poiseuille law).
5. Calculate the dynamic viscosity based on the Hagen-Poiseuille law and on the known geometry.
6. Calculate the Reynolds number for the different tube diameters based on the flow velocities and make a statement concerning the flow characteristics of the different tubes.

## Set-up and procedure

### Set-up

#### Experiment set-up

Figure 1 shows the complete experiment set-up. The following section describes the individual steps and provides information concerning the start-up of the various devices. We suggest that the experiment is set up by lab course staff since first-time set-up is very time-consuming and error-prone.

#### Layout of the flow system:

The flow system consists of three rigid flow tubes with different diameters. They are connected among each other with two flexible tubes (length 30 cm, inner diameter 10 mm). The branches for the connection to the vertical tubes for the pressure measurement are located in the middle of these pieces of flexible tubing. The connection to the pump is realised with two additional tubes (inner diameter 10 mm). The tube with the branch (filling connector) must be connected to the suction side of the pump (central connector). The longer tube with the branch towards the vertical tube near the first measurement tube (outer diameter 20 mm) must be connected to the delivery side (tangential connector) of the pump. The order of the tubes is as follows in the direction of flow: tube 1 (outer diameter 20 mm, wall 2 mm), tube 2 (outer diameter 15 mm, wall 2.5 mm), and tube 3 (outer diameter 10 mm, wall 1.5 mm). The four vertical tubes are connected to the tubes with Luer connectors from left to right in the order of the direction of flow (see also the setup diagram). A support rod (diameter approximately 12 mm) with a foot is used to hold the pressure scale.

**Note:** Align the pressure scale in plumb when there is no flow in order to ensure that all of the scales display the same value.

**Attention:** In order to ensure a tight connection, the points of connection of the flexible tubes to the rigid tubes and pump connectors should be secured with cable ties (included in the scope of supply). Lay the tubes so that they do not kink in order to avoid any additional flow resistance.

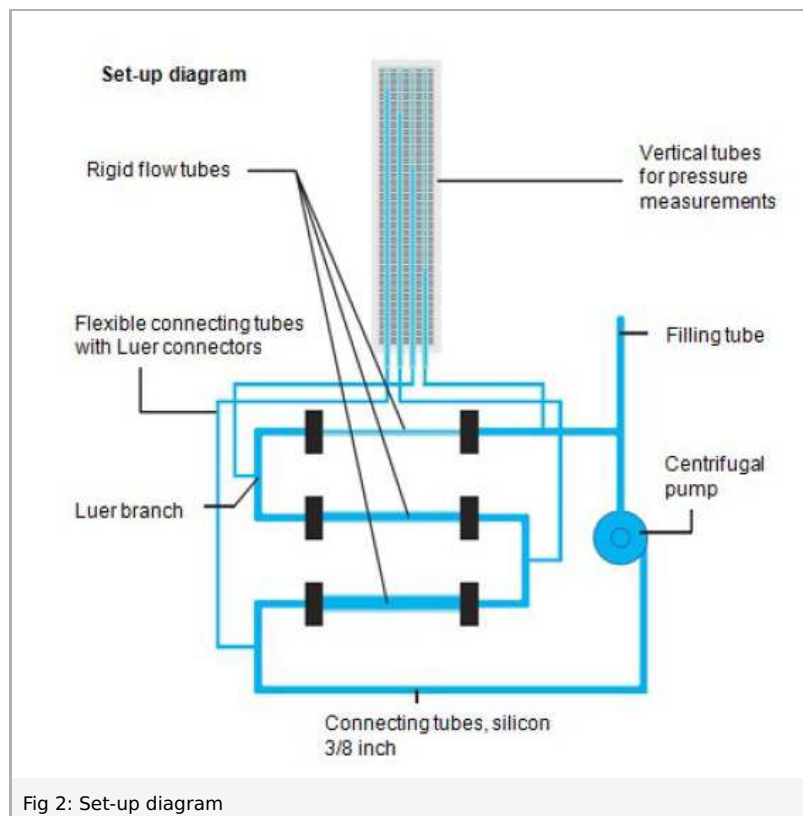


Fig 2: Set-up diagram

#### Filling the rigid and flexible tubes

Prior to filling the tubes, mix the Doppler phantom liquid thoroughly. The liquid includes scattering particles for measuring the ultrasound scatter signal. These particles have a higher density than the liquid and, therefore, tend to settle at the bottom of the bottle. For mixing, turn the bottle upside down and swirl it gently until the deposit (scattering particles) has become detached. Then, shake the bottle vigorously in order to mix the content completely. Whenever the tube system needs to be topped up, the liquid must be mixed again.

Fill the system via the filling tube and with the aid of the funnel that is included in the scope of supply. Insert the funnel only loosely into the flexible tube in order to allow the air in the system to escape. For ease of handling, the tube for filling the system

should also be held in a support system. Fill the liquid in slowly so that the air can escape more easily. Swirl the bottle repeatedly in between in order to shake up the scattering particles. Wait a moment after filling until all of the air bubbles have risen in the tubes. Air bubbles produce a very strong ultrasound signal, which may cause the Doppler measuring device to overshoot and, thereby, to distort the measurement result. If necessary, vent the pump head by tilting the pump. In order to make optimum use of the pressure scale, fill the vertical tubes approximately by half (scale value approximately 50 to 55 cm). Then, the filling tube can be sealed with a plug (included in the scope of supply). Further venting is realised by slow pumping via the vertical tubes.

**Attention:** Prior to starting the pump, ensure that the speed control of the centrifugal pump is turned anti-clockwise up to the stop. In the case of high flow velocities, the pressure in the tube system increases strongly so that the liquid column of the pressure scale may exceed its maximum and liquid may escape. Limit the maximum pump speed for the half-filled system to approximately 50 % or connect the upper ends of the vertical tubes with flexible silicon tubes.

### Determination of the static pressure

For the pressure measurements, the scale value is read off the vertical tubes for every different flow setting. This value corresponds to the static pressure with the unit "cm liquid column". Deposits of scattering particles may lead to differences in concentration in the vertical tubes. Due to the osmotic pressure, this will cause different liquid levels in the tubes, which must be determined and subtracted accordingly prior to the pressure measurement (without flow).

**Note:** The pressure measuring points are located at points of identical tube diameters and, therefore, for laminar flows with identical flow velocity in order to be able to measure the pressure drop across the measuring points directly via the change in static pressure.

**Attention:** The rigid flow tubes and the set of flexible tubes can be cleaned with commercially available cleaning agents. In order to remove gel residues, however, you can simply use water with a little washing-up liquid and, if necessary, a soft cloth. If the tubes have not been used for a longer period of time, the scattering particles of the ultrasound phantom liquid will settle to the bottom of the tubes and may flocculate. These flakes, however, can be dissolved again after a certain period of quick pumping and by turning the tubes.

**Note:** If the filled circuit is left to stand for a longer period of time, we recommend sealing the vertical tubes in order to prevent the water in the liquid from evaporating.

### Attaching the Doppler prisms:

The flow velocity is measured with the "FlowDop" Doppler ultrasound unit and the Doppler prisms. For this purpose, the Doppler prism is attached to the tube that is to be examined. The point of attachment should not be located directly behind the tube inlet, since there might be swirls and turbulences in the flow, which in turn may affect the measurement results. For tubes with a diameter of 10 and 15 mm, a measurement in the area of the middle of the tube up to the tube end, and for the tube with an outer diameter of 20 mm, a measurement at the tube end (flow outlet) will provide the best results. Prior to attaching the prism, a layer of ultrasound gel must be applied to the curved surface. This is necessary in order to achieve good acoustic coupling between the prism and tube. Attach the prism to the tube and turn and move it slightly in order to distribute the gel layer evenly and without bubbles.

In the next step, the surface of the ultrasound transducer and the angular surfaces of the prism are also treated with gel. There should always be a sufficient amount of gel between the transducer and the prism during the measurement in order to achieve sufficient signal intensities. Use the 30° surface of the Doppler prism for measuring the flow velocity (best ratio between the signal intensity and the Doppler angle for all of the tube diameters and flow rates that are used for this experiment).

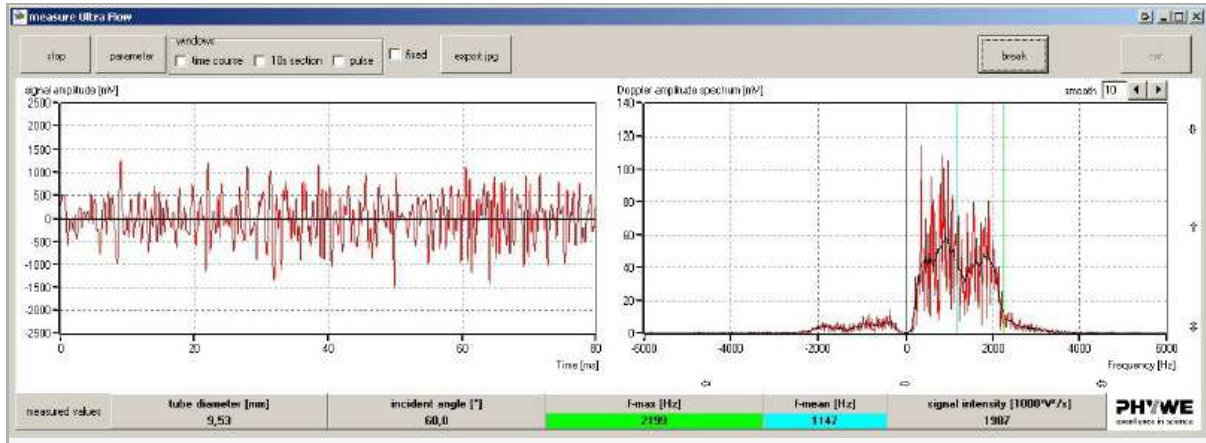
### Doppler ultrasound unit:

The measurement uses a 2 MHz ultrasonic probe, which must be connected to the Doppler unit. Set the transducer frequency of 2 MHz with the aid of the frequency selector switch. The setting of the "Sample Volume" switch is very important. For the measurement of the mean flow velocity, this switch must be set to "Large" (in the case of "Small", only signals coming from a small part of the tube will be measured). The transmission power can be varied with the "Power" controller, whereas "Gain" controls the reception gain. "Power" and "Gain" should always be set so that the signal intensity that is displayed by the software is high enough without overshooting. If the Doppler liquid is well mixed and if there are enough scattering particles in the measurement volume, we recommend using the settings "Power"=HIGH and "Gain"=20 dB. Please avoid changing the settings during a measurement. The volume of the sound signal of the measurement can be controlled with the "Audio Volume" controller.

**Note:** If the signal amplitude decreases strongly during the measurement, ensure that too many scattering particles have not settled to the bottom (mix the liquid again). In order to obtain a Doppler frequency spectrum that corresponds to the mean flow velocity, it must be ensured that there are enough scattering particles in the measuring area of the tube and that these particles are distributed evenly.

**Note:** Set the gain of the Doppler unit so that the signal does not overshoot, which would otherwise result in errors of the frequency determination.

### Software



The software reads the measurement data out of the Doppler ultrasound unit and presents them in graphical form. The left window shows the current scattering intensities, while the window on the right-hand side shows the spectrum of these data. The spectrum is used to determine and display the two frequency values  $f_{\text{mean}}$  and  $f_{\text{max}}$ . If the geometric conditions are known, the mean and maximum flow velocity can be calculated based on these values and in accordance with the Doppler law.

**Attention:** Please comply with the special operating and safety instructions in the operating manuals of the Doppler ultrasound unit and centrifugal pump.

## Procedure

Adjust four different flow velocities in % at the centrifugal pump (e.g. 20 %, 30 %, 40 %, and 50 %). For each velocity, measure the mean and maximum frequency shifts  $f_{\text{max}}$  and  $f_{\text{mean}}$  for all of the three rigid tubes with the Doppler ultrasound unit and read the values off the vertical tubes.

**Note:** Wait some time in order to measure under steady flow conditions (no changes of the pressure scale).

## Evaluation and result

### Theory

### Results

Table 1: Geometries

	$d$ (outer)	wall	$d$ ( inner)	Cross-section
	mm	mm	mm	mm <sup>2</sup>
Tube 1	20.0	2.0	16.0	201.1
Tube 2	15.0	2.5	10.0	78.5
Tube 3	10.0	1.5	7.0	38.5

Tube length: 300 mm

Length of the intermediate pieces of flexible tubing: 300 mm (150 mm each from the rigid tube to the pressure measurement branch)

Inner diameter (intermediate piece) Approx.: 10 mm

#### Characteristics of the Doppler liquid:

Sound velocity: 1800 m/s

Viscosity: 12 mPa/s

Density: 1.15 g/cm<sup>3</sup>

#### Characteristics of the Doppler measurement:

Prism sound velocity: 2700 m/s

Doppler frequency: 2.0 MHz

Irradiation angle: 30°

#### Doppler angle calculated with equation (3):

Table 2: Angle of irradiation into the liquid: 19.5° Doppler angle: 70.5°

Pump	Tube 1		Tube 2		Tube 3	
	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]	[Hz]
20	110	60	200	110	400	220
30	210	120	460	250	950	510
40	325	180	690	390	1450	800
50	420	240	990	540	2020	1110

Table 3: Static pressure measurement values:

Pump	Pressure	Pressure	Pressure	Pressure
[%]	p1	p2	p3	p4
	[cm liq. column]	[cm liq. column]	[cm liq. column]	[cm liq. column]
20	58.8	57.0	54.4	47.8
30	66.0	60.8	54.0	37.2
40	76.3	66.4	53.5	21.3
50	89.3	73.3	53.0	1.8

#### Flow velocities (calculated based on the Doppler law (2)):

Table 4

Pump	Tube 1			Tube 2			Tube 3		
[%]	$v_{max}$	$v_{mean}$	$v_{max}/v_{mean}$	$v_{max}$	$v_{mean}$	$v_{max}/v_{mean}$	$v_{max}$	$v_{mean}$	$v_{max}/v_{mean}$
	[cm/s]	[cm/s]		[cm/s]	[cm/s]		[cm/s]	[cm/s]	
20	14.9	8.1	1.83	27.0	14.9	1.82	54.0	29.7	1.82
30	28.4	16.2	1.75	62.1	33.8	1.84	128.3	68.9	1.86
40	43.9	24.3	1.81	93.2	52.7	1.77	195.8	108.0	1.81
50	56.7	32.4	1.75	133.7	72.9	1.83	272.7	149.9	1.82

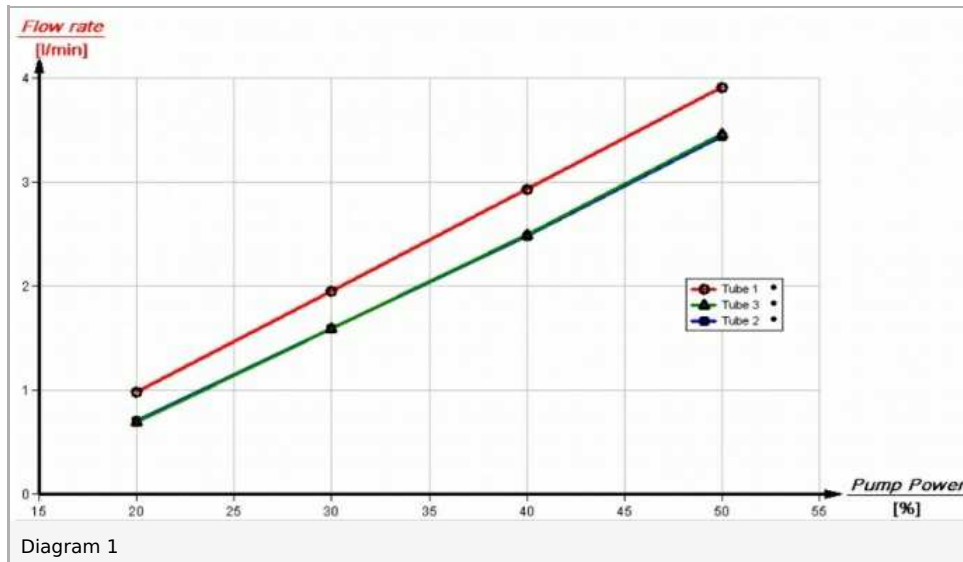
The smaller the tube diameter is, the higher the flow velocity. This corresponds to the law of continuity. The ratios between the mean and maximum flow velocity are in the range of 1.8 for all of the measurements and, therefore, are close to the ratio of the laminar tube flow of 2. The Doppler scattering method is used to determine the velocity of the particles in the liquid and not of the liquid itself. Since - in the ideal case - the maximum flow velocity in the middle of the tube is represented by only one scattering particle at maximum, it cannot be determined with the Doppler scattering method. One requires a scatter signal that exceeds the electronic noise and also several scattering particles that move at close to the maximum velocity. As a result, it will always be under-determined. Since the ratio does not increase even if the pump power (speed) increases, a transition to a turbulent flow cannot be measured within the measuring range of the tubes.

**Flow rates (calculated based on the continuity equation (4)):**

Table 5

Pump	Tube 1	Tube 2	Tube 3	Q
[%]	Q	Q	Q	Mean R2 R3
	[l/min]	[l/min]	[l/min]	[l/min]
20	0.98	0.70	0.69	0.69
30	1.95	1.59	1.59	1.59
40	2.93	2.48	2.49	2.49
50	3.91	3.44	3.46	3.45

The flow rates that were calculated based on the cross-sections and mean flow velocities are shown in diagram 1.



According to the continuity equation, the flow rates should be identical for all of the tube diameters with the same pump settings. This, however, is only the case for tube 2 and tube 3. In the case of tube 1 (largest diameter), the flow that is calculated from the mean velocity and tube cross-section is too high. This is due to the fact that the flow velocity is determined too high. This error occurs regardless of the pump rate and Doppler frequency. The reason for this could be the inertia of the scattering particles. They come out of the connecting tube at a higher velocity (smaller diameter) and do not adapt to the liquid velocity in the rigid tube over the length of 30 cm. This can be proven by Doppler frequency measurements along the tube cross-section. The Doppler ultrasound unit measures the velocity of the scattering particles.

In addition, the mean of the flow rates that were measured at tube 2 and tube 3 is used as the basis for the calculation.

The static pressure measurements at the measuring points 1 to 4 are used to determine pressure differences  $\Delta p$  (pressure drops) across the rigid tubes with connecting pieces.

**Pressure differences between the measuring points**

Table 6

Q	1-2	2-3	3-4
[l/min]	$\Delta p$	$\Delta p$	$\Delta p$
	[mm liq. column]	[mm liq. column]	[mm liq. column]
0.69	18	26	66
1.59	52	68	168
2.49	99	129	322
3.45	160	203	512

For the determination of the pressure drop across the rigid tube sections, it is assumed that the pressure drop across the flexible tube sections with a connection to the vertical tubes (measuring sections) corresponds approximately to the pressure drop across tube 2 due to the similar geometric conditions, i.e.:

$$\Delta p(\text{tube connector}) = \Delta p(\text{tube 2}) \frac{\Delta p(2-3)}{2} \quad (10)$$

This leads to the following pressure drop across the tubes:

**Pressure drop across the tubes:**

Table 7

Q	Measuring section	Tube 1	Tube 2	Tube 3
[l/min]	$\Delta p$	$\Delta p$	$\Delta p$	$\Delta p$
	[mm liq. column]	[mm liq. column]	[mm liq. column]	[mm liq. column]
0.69	13.0	5.0	13.0	53.0
1.59	34.0	18.0	34.0	134.0
2.49	64.5	34.5	64.5	257.5
3.45	101.5	58.5	101.5	410.5

The density of the liquid can be used to calculate the pressure drop in SI units (Pa) in accordance with equation (9):

**Pressure drop across the tubes in Pascal:**

Table 8

Q	Measuring section	Tube 1	Tube 2	Tube 3
[l/min]	$\Delta p$	$\Delta p$	$\Delta p$	$\Delta p$
	[Pa]	[Pa]	[Pa]	[Pa]
0.69	147	56	147	598
1.59	384	203	384	1512
2.49	728	389	728	2905
3.45	1145	660	1145	4631

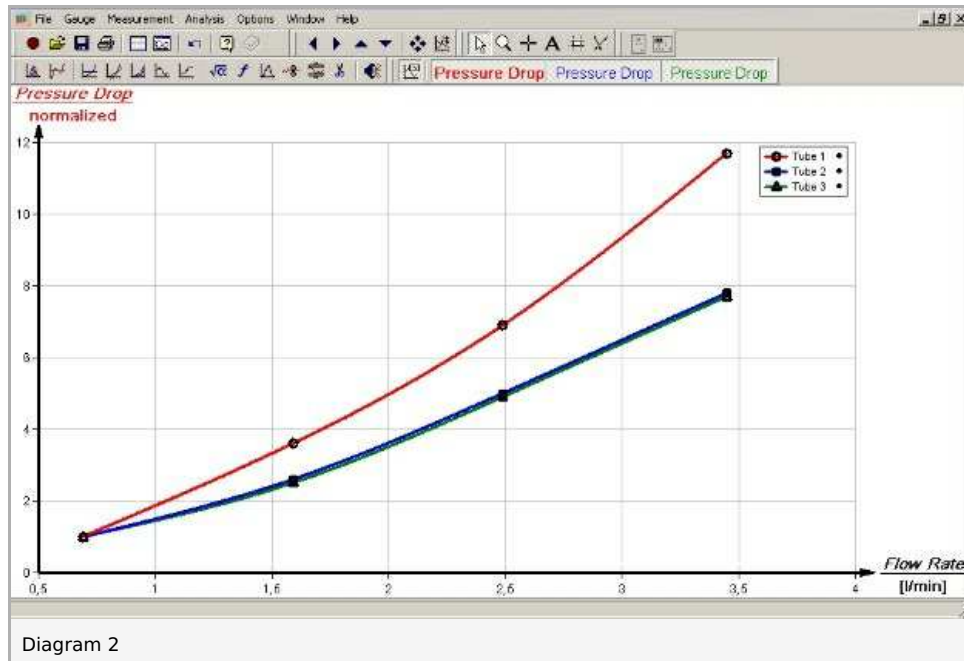
For laminar flows, the pressure drop would have to increase linearly in dependence on the flow rate (flow velocity). In order to examine this dependence for the various tube sections, the pressure drops are normalised with regard to the lowest flow rate and are represented in diagram 2.

**Pressure drop across the tubes (normalised):**

Table 9

Q	Tube 1	Tube 2	Tube 3
[l/min]	$\Delta p$	$\Delta p$	$\Delta p$
	[Pa]	[Pa]	[Pa]
0.69	1.0	1.0	1.0
1.59	3.6	2.6	2.5
2.49	6.9	5.0	4.9
3.45	11.7	7.8	7.7





The pressure drop is not really linear for any of the tube sections. Since the changes in tube 2 and 3 are identical regardless of the different diameters and flow velocities, there are probably additional pressure losses at the tube branches for the pressure measuring points. Tube 1, on the other hand, has turbulent pressure losses where the flow coming from the flexible tube enters the rigid tube. This can also be observed based on the behaviour of the scattering particles when the pump rate changes (partial return flows). As a result, the following resistances of the tube sections that were calculated with equation (6) and, therefore, also the viscosities that were calculated with equation (7) contain errors, especially for tube 1 (the values are too high).

**Tube resistance**

Table 10

Flow rate	Tube 1		Tube 2		Tube 3	
	$\Delta p$	R	$\Delta p$	R	$\Delta p$	R
0.69	56	4.98	147	12.70	598	51.78
1.59	203	7.66	384	14.47	1512	57.04
2.49	389	9.39	728	17.55	2905	70.07
3.45	660	11.49	1145	19.93	4631	80.59

**Dependence of the tube resistance on the diameters**

Table 11

Q[l/min]	0.69	1.59	2.49	3.45
d	R	R	R	R
[mm]	[MPa s/m <sup>3</sup> ]	[MPa s/m <sup>3</sup> ]	[MPa s/m <sup>3</sup> ]	[MPa s/m <sup>3</sup> ]
7.0	51.8	57.0	70.1	80.6
10.0	12.7	14.5	17.6	19.9
16.0	4.9	7.7	9.4	11.5

The dependence of the resistance on the tube diameter in accordance with (7a) is shown in diagram 3.

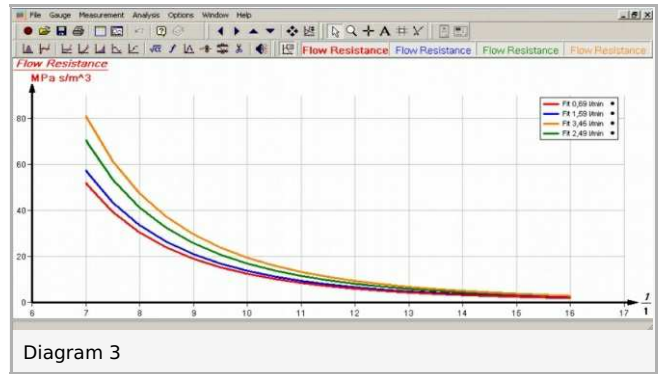
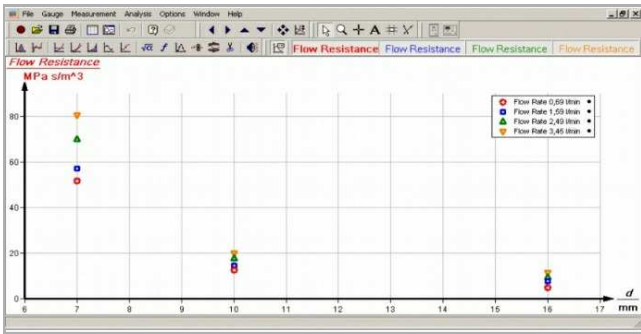


Diagram 3

Viscosity of the Doppler liquid:

Table 12

	Tube 1			Tube 2			Tube 3		
Flow rate	d	R	$\eta$	d	R	$\eta$	d	R	$\eta$
[l/min]	[mm]	[MPas/m³]	[MPa s]	[mm]	[MPas/m³]	[MPa s]	[mm]	[MPas/m³]	[MPa s]
0.69	16.0	4.9	26.2	10.0	12.7	10.4	7.0	51.8	10.2
1.59	16.0	7.7	41.1	10.0	14.5	11.8	7.0	57.0	11.2
2.49	16.0	9.4	50.3	10.0	17.6	14.4	7.0	70.1	13.8
3.45	16.0	11.5	61.6	10.0	19.9	16.3	7.0	80.6	15.8

In the case of tube 2 and tube 3, the viscosity for lower flow rates matches very well the viscosity that was determined rheologically (12mPa s). As far as tube:2 is concerned, the conditions for the law of Hagen-Poiseuille are not fulfilled (see the error concerning the resistance determination that is discussed above). Table 13 shows the Reynolds numbers for the various tube sections that were determined with the aid of equation (6).

Reynolds number

Table 13

Pump	Tube 1		Tube 2		Tube 3	
	v[cm/s]	Re	v[cm/s]	Re	v[cm/s]	Re
20	8.1	57	14.9	134	29.7	342
30	16.2	73	33.8	268	68.9	792
40	24.3	89	52.7	345	108.0	1242
50	32.4	97	72.9	421	149.9	1723

For all of the tubes and flow velocities, the Reynolds numbers are below 2300 and, therefore, also below the critical value. At these flow rates, the additional turbulent losses in the circuit do not occur in the tube sections themselves, but mainly at the junction and branch points.