Principle

When a body is affected by a constant force, it will undergo a constant acceleration. The change of motion is proportional to the accelerating force. In this experiment, the relation between the acceleration of a cart on the demonstration track and its mass as well as the relation between its acceleration and the accelerating force will be studied.

Related topics

Newton's laws of motion, change of motion due to force, inertial mass, velocity-time equation.

Learning objective

According to Newton's second law the change of motion of an object is proportional to the affecting force:

$$\vec{F} = m \cdot \frac{\mathrm{d}^2 \vec{x}}{\mathrm{d}t^2} = m \cdot \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = m \cdot \vec{a} \quad .$$

The relation between mass and acceleration will be investigated for various inertial masses and different accelerating forces.

Tasks

1. Determination of the cart acceleration as a function of the accelerated mass
2. Determination of the acceleration as a function of the force.

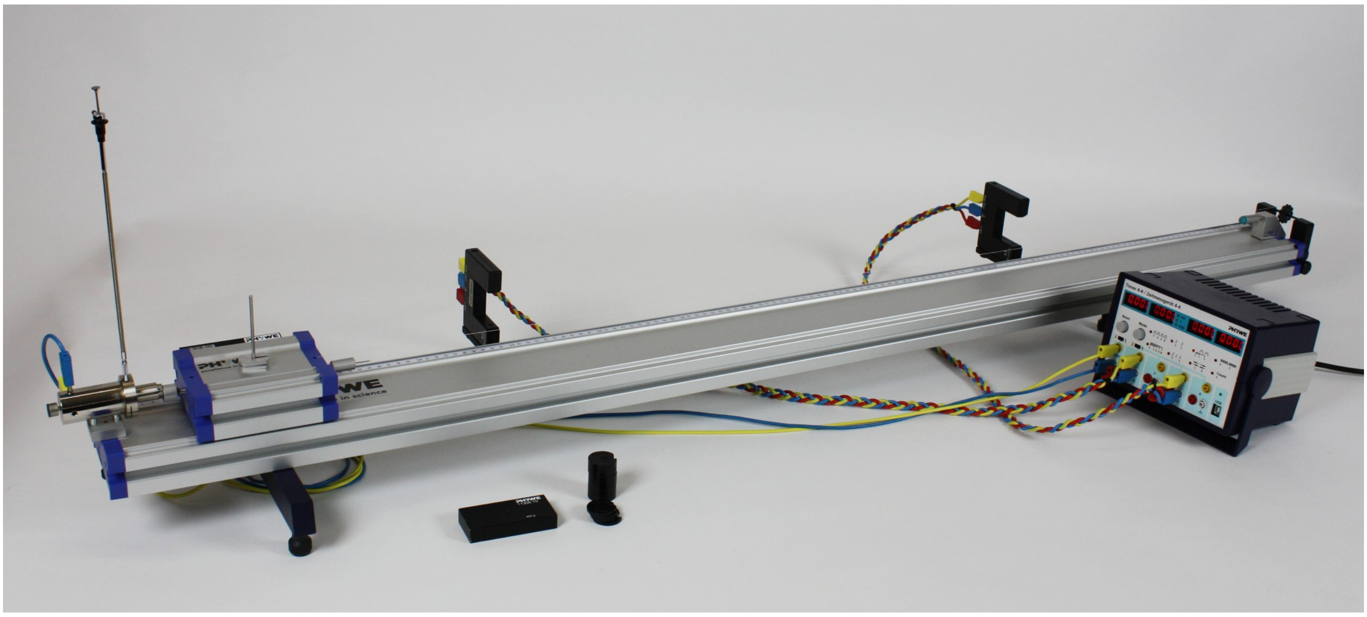

Fig. 1: Experimental set-up

Equipment

1	Demonstration track, aluminium, l = 1.5 m	11305-00
1	Cart, low friction sapphire bearings	11306-00
1	Shutter plate for low friction cart, width: 100 mm	11308-00
1	Needle with plug	11202-06
1	Tube with plug	11202-05
1	Plasticine, 10 sticks	03935-03
1	Magnet with plug for starter system	11202-14
2	Light barrier, compact	11207-20
2	Holder for light barrier	11307-00
1	Timer 4 – 4	13604-99
2	Connecting cord, 32 A, 1000 mm, red	07363-01
3	Connecting cord, 32 A, 1000 mm, yellow	07363-02
3	Connecting cord, 32 A, 1000 mm, blue	07363-04
1	Starter system for demonstration track	11309-00
1	End holder for demonstration track	11305-12
1	Weight for low friction cart, 400 g	11306-10
1	Weight holder, silver bronze, 1 g	02407-00
4	Slotted weight, black, 10 g	02205-01
3	Slotted weight, black, 50 g	02206-01
20	Slotted weight, blank, 1 g	03916-00
1	Silk thread, l = 200 m	02412-00
1	Holder for pulley	11305-11
1	Pulley for demonstration track	11305-10
1	Portable Balance, OHAUS CS2000	48917-93

Optional:

1	Pulley, movable, diameter: 40 mm, with hook	03970-00

Set-up

The experimental set-up is shown in Fig 1:

1. To compensate marginal friction effects, the demonstration track has to be adjusted by the set-screws at the track bases. Turn the screws so that you cause a slight inclination of the track but avoid that the cart will start to roll rightwards.
2. Mount the starter system on the left end of the track. The plunger has to point away from the cart so it can be released without an initial momentum (see Fig. 2).
3. Place the end holder at the far end of the track and insert the tube filled with Plasticine. (see Fig. 3) This will prevent a hard collision and the cart will brake gently.
4. The pulley for demonstration track has to be attached to the right end of the track by the holder with the incremental wheel inserted.
5. Assemble the cart with the magnet to the left side and the shutter plate.
6. The thread end is set in the right vertical drilling of the cart and fixed due to the insertion of the needle (see Fig. 4).

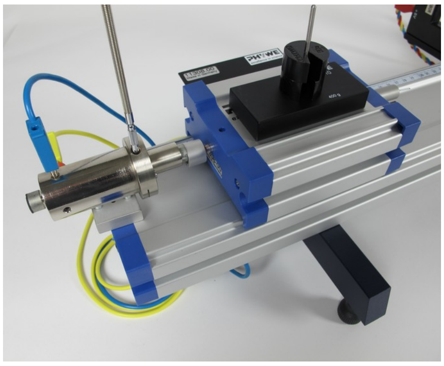

Fig. 2: Starter system without initial momentum

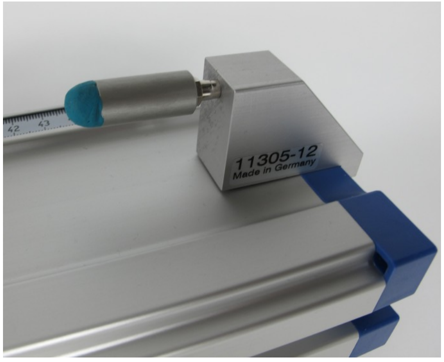

Fig. 3: End holder with Plasticine

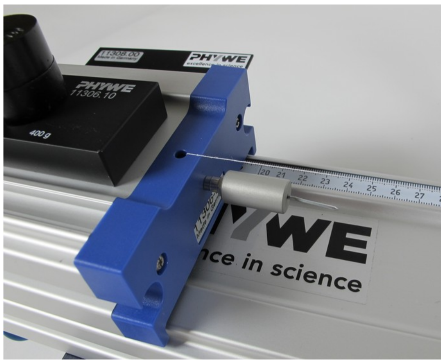

Fig. 4: Attachment of the silk thread to the cart

7. The silk thread is put over the incremental wheel of the pulley and knotted to the weight holder, which should hang freely directly below the pulley (see Fig. 5). The weight holder including 5 to 20 slotted weights (1 g each) serves as a constant accelerating force. Please note that the thread always runs parallel to the track.
8. The mass of the cart can be adjusted using the black lacquered weights.
9. Two light barriers are mounted on the track by the holders in an even distribution. Make sure that both light barriers can be passed by the rear part of the shutter plate when rolling the cart (see Fig. 6).
10. The light barriers have to be connected to the jacks in the timer panels "1" and "3". Therefore, the yellow light barrier jacks are connected to the yellow jacks of the timer, the red jacks to the red jacks, and the blue jacks of the light barriers to the white jacks of the timer (see Fig. 7).
11. The starter system is connected to the "Start" socket of the timer. Be aware of the proper polarity, the red jack of the starter system has to be connected to the yellow jack of the timer.
12. The two slide switches of the timer set the trigger edges and are moved to the right position for "falling edge" (↓).

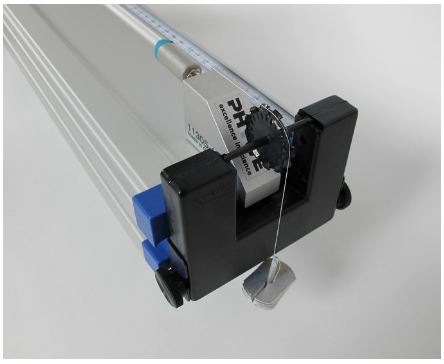

Fig. 5: Initial position of the weight holder

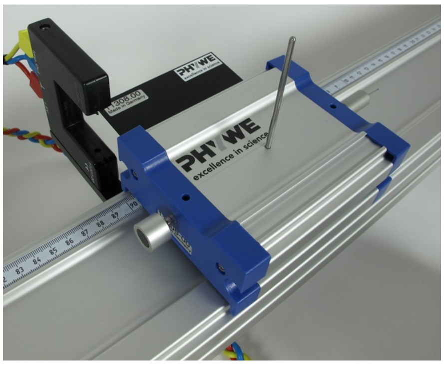

Fig. 6: Passing of a light barrier

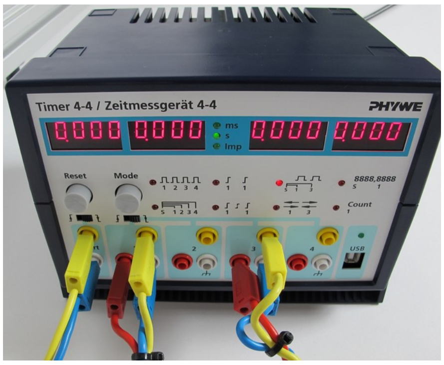

Fig. 7: Connection of the light barriers and the starter system

Procedure

1. In the experiment, the cart will be forced to a uniform accelerated motion due to the descent of the weight holder. The acceleration of the cart has to be observed regarding the total inertial mass and the accelerating mass, as well. Therefore, the mass of the assembled cart (but without any of the black lacquered additional weights) has to be determined by use of the balance.
2. The cart is released from the starter system and is forced to a constant acceleration until the weight holder reaches the floor.
3. To determine the acceleration of the cart, the measurement is performed in timer mode 5 ( ). In this set-up, both light barriers measure two times at once. The elapsed time t_i from the start until the cart reaches each of the light barriers is shown in the displays 1 and 3 of the timer, whereas the interruption time Δt_i during the passing of the shutter plate through the optical path of a light barrier is displayed in screens 2 and 4.
4. The velocity-time equation gives us the acceleration of the cart:

   $$v(t) = a \cdot t \Leftrightarrow a = \frac{v(t)}{t} = \frac{w / \Delta t}{t}$$

   using a shutter plate width w = 100 mm.
5. To evaluate the acceleration as a function of the mass of the cart, a whole series of measurements has to be carried out, where the mass of the cart is increased in steps of about 10 g to 50 g. Meanwhile, the mass of the weight holder has to remain constant.
6. In the second part of the experiment, the acceleration should be measured as a function of the accelerating force. Therefore, a second series of measurements has to be performed, at which the total mass of the system remains constant while the accelerating force F_G is altered. That can be achieved by a mass transfer from the cart to the weight holder. It is recommended to initially place about ten slotted weights of 1 g on the cart (see Fig. 8). For each measurement, gradually transfer one of the weights from the cart to the weight holder. The accelerating mass should not exceed a maximum of 20 g.

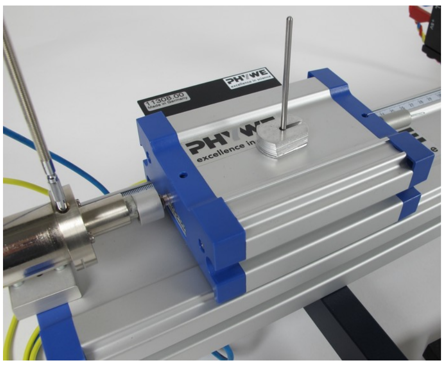

Fig. 8: Additional weights for the measurement of force

Observation

Increasing the inertial mass for a constant accelerating force causes a decelerated motion of the cart. In the second series of measurements, a mass transfer causes a change of acceleration. The total mass of the system is constant, but in fact, the mass of the weight holder is accountable for the acceleration of the cart, exclusively.

Evaluation

a) Acceleration as a function of the total inertial mass

1. A measurement example for the determination of the dependency of inertial mass and acceleration is provided in Tab. 1. The total mass is given by the mass of the cart M, consisting of the empty mass M_0 of the cart and the mass M_{sw} of the additional slotted weights, as well as the constant mass m of the weight holder including the slotted weights.
2. The measured times t_i and Δt_i yield the acceleration of the cart a_i, instantaneously, using the velocity-time equation and the shutter plate width w. Within the limits of the measurement accuracy, the acceleration is expected to be identical for both light barriers thus the mean acceleration

   $$a_\mathrm{m} = \frac{a_1 + a_2}{2} = \frac{w(t_1 \cdot \Delta t_1 + t_2 \cdot \Delta t_2)}{2}$$

   should be used for the further approach.
3. Fig. 9 shows the mean acceleration a_m, caused by the dropping weight holder mass m = 10 g, as a function of the total inertial mass $M+m$. According to the results, increasing masses correspond to a decrease of the cart acceleration.
4. Fig. 10 shows the same measurement, but, this time, the acceleration is plotted against the reciprocal value $1/(M+m)$ of the total mass. The diagram reveals a linear dependency which is in agreement with Newton's second law of motion. The primal force causing the motion is the gravitational force

   $$F_\mathrm{G} = m \cdot g \quad ,$$

   which consists of the gravitational acceleration g and the mass of the weight holder m.
5. Due to the fact that the weight holder is directly connected to the cart, the gravitational force is equal to the force F_mot which causes the motion of the entire mass system:

   $$F_\mathrm{G} = m \cdot g = (M + m) \cdot a = F_\mathrm{mot} \quad .$$

   That results in the linearity between acceleration and the reciprocal mass as shown in Fig. 10:

   $$a = \frac{m \cdot g}{M + m} \propto \frac{1}{M + m} \quad .$$

   The evaluation of the presented measurement example gives a slope of 0.0946 kg·m/s² = 0.0946 N which is in a very good aggreement with the theoretical prediction of $m \cdot g$ = 0.0981 N.

Table 1: Measurement example with cart mass M_0 = 397 g, additional slotted weights M_{sw}, total cart mass $M = M_0+M_{sw}$, constant accelerating mass m = 10 g, shutter plate width w = 0.100 m, and mean acceleration a_m.

M_{sw} in kg	$M+m$ in kg	t_1 in s	Δt_1 in s	t_2 in s	Δt_2 in s	$1/(M+m)$ in kg	a_m in m/s²
0.010	0.417	2.085	0.208	2.758	0.160	2.40	0.229
0.030	0.437	2.143	0.213	2.831	0.165	2.29	0.217
0.050	0.457	2.210	0.218	2.904	0.168	2.19	0.206
0.070	0.477	2.237	0.224	2.958	0.172	2.10	0.198
0.090	0.497	2.290	0.228	3.025	0.175	2.01	0.190
0.110	0.517	2.318	0.232	3.067	0.179	1.93	0.184
0.150	0.557	2.410	0.242	3.190	0.186	1.80	0.170
0.190	0.597	2.479	0.251	3.288	0.193	1.68	0.159
0.400	0.807	2.922	0.292	3.866	0.226	1.24	0.116
0.500	0.907	3.068	0.312	4.074	0.241	1.10	0.103
0.560	0.967	3.225	0.324	4.271	0.250	1.03	0.095

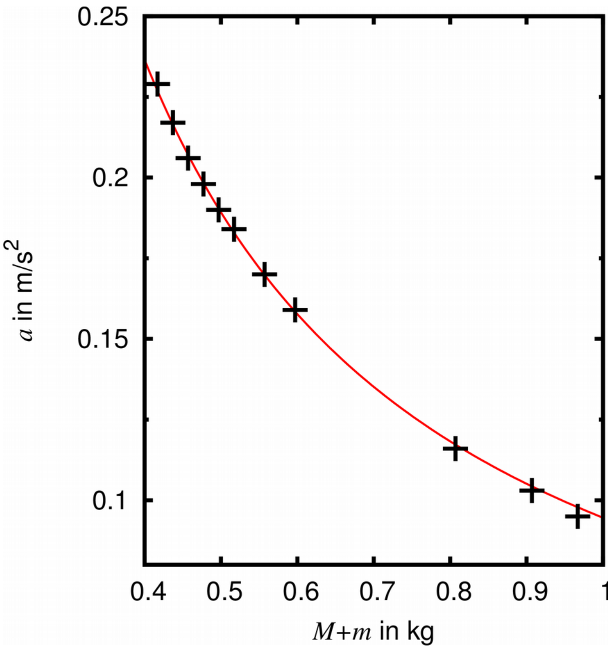

Fig. 9: Dependency of the cart acceleration on the total mass for a constant accelerating mass m = 10 g

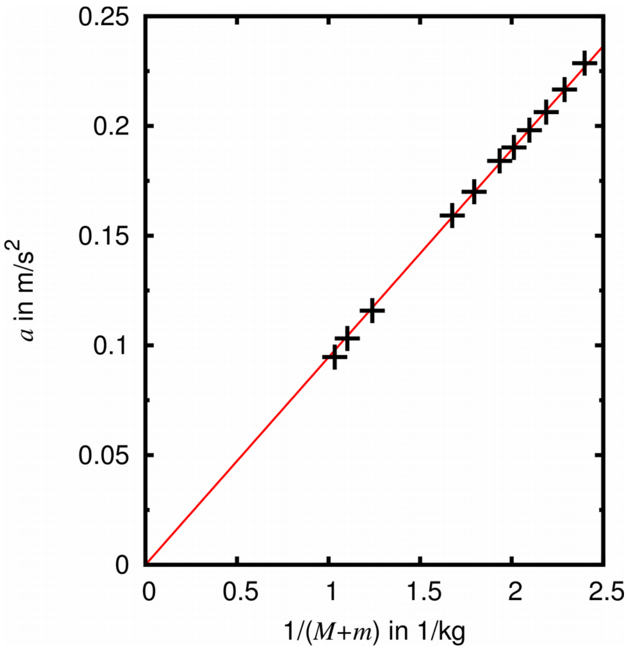

Fig. 10: Dependency of the cart acceleration on the reciprocal total mass for a constant accelerating mass m = 10 g

b) Acceleration as a function of force

1. Table 2 shows a measurement example for the dependency of the acceleration on the accelerating mass. While the total mass $M+m$ remains constant, the mass of the weight holder m is stepwise adjusted.

Table 2: Measurement example for a constant total mass $M+m$ = 967 g, variable accelerating mass m, shutter plate width w = 0.100 m, and mean acceleration a_m.

m in kg	t_1 in s	Δt_1 in s	t_2 in s	Δt_2 in s	$F_G = m \cdot g$ in kg·m/s²	a_m in m/s²
0.010	3.225	0.324	4.271	0.250	0.098	0.095
0.011	3.025	0.309	4.022	0.239	0.108	0.106
0.012	2.847	0.295	3.799	0.228	0.118	0.117
0.013	2.766	0.284	3.681	0.219	0.128	0.126
0.014	2.677	0.273	3.558	0.210	0.137	0.135
0.015	2.555	0.265	3.406	0.203	0.147	0.146
0.016	2.481	0.258	3.310	0.198	0.157	0.154
0.017	2.386	0.249	3.188	0.191	0.167	0.166
0.018	2.350	0.242	3.129	0.185	0.177	0.174
0.019	2.281	0.234	3.055	0.180	0.186	0.185
0.020	2.237	0.230	2.976	0.177	0.196	0.192

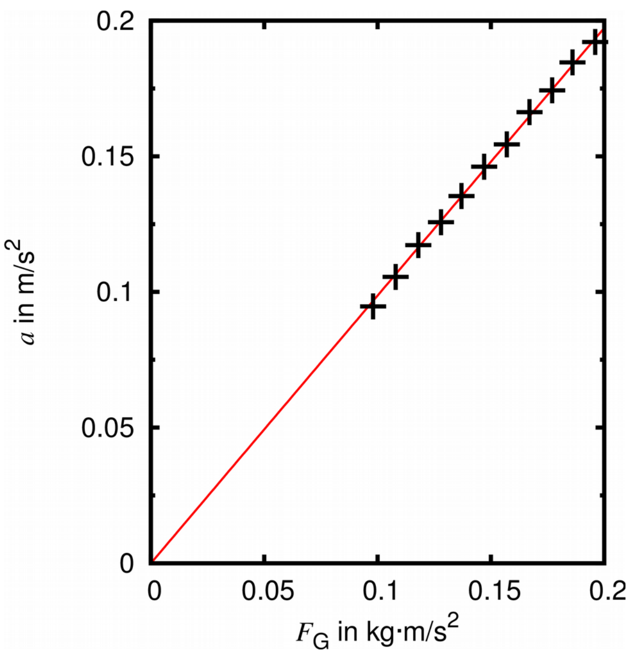

Fig. 11: Dependency of the acceleration on the accelerating force $F_G = m \cdot g$

2. From the balance of forces also follows a linearity between acceleration and the gravitational force

$$a = \frac{F_G}{M+m} \propto F_G \ ,$$

that is shown in Fig. 11. The slope of the measurement example yields a proportionality factor of 0.986 kg^{-1} which is equivalent to an inertial mass of 1.014 kg. In comparison to the actual total mass of 0.967 kg, that has been determined by the weighing, there is only a slight deviation of less than 5 %.

Remarks

1. To reduce the distance between weight holder and incremental wheel, the thread length can be shortened by multiple twists of the needle with plug in the frontal drilling of the cart. This procedure will spool the thread inside of the cart and therefore lift the weight holder.
2. The light barrier interruption times Δt_i have been used to determine the instantaneous velocities of the cart. However, the cart is still accelerating during the interruption process so the calculated instantaneous velocities actually represent the mean velocity during that period. Considering that fact, minor deviations in a range of up to a few percent from the theoretical values can be explained.
3. If the height of the table is not sufficent to accelerate the cart across the entire track, it is possible to double the distance by use of movable pulley as shown in Fig. 12. The guidance bracket that comes along with the holder for pulley has to be attached to the screws, as well. Then, the end of the silk thread must not be fixed to the weight holder but to the bracket, instead. The movable pulley is placed on top of the thread and, finally, the weight holder including the adjustable slotted weights is mounted to the hook of the movable pulley.

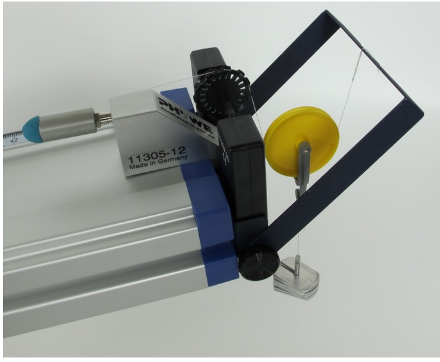

Fig. 12: Usage of a movable pulley to increase the acceleration distance

Please notice that only half of the gravitational force F_G is invested in the acceleration of the cart, as the distance covered by the cart is now twice as long as that of the weight holder.

4. The entire experiment solely treats one-dimensional motion thus as a simplification, the dependency on the direction has been neglected. Generally, one must consider that force, acceleration, velocity, and displacement are vectors.