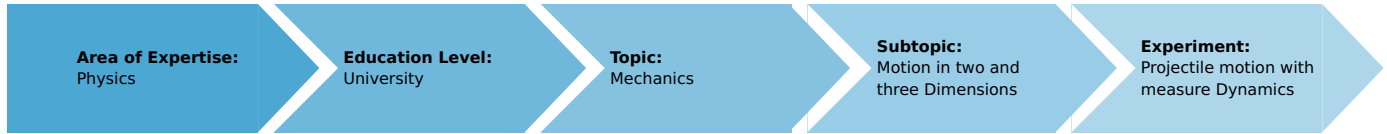


# Projectile motion with measure Dynamics (Item No.: P2131180)

## Curricular Relevance



### Difficulty



Easy

### Preparation Time



10 Minutes

### Execution Time



20 Minutes

### Recommended Group Size



2 Students

### Additional Requirements:

- video camera

### Experiment Variations:

- P2131100 Projectile motion

### Keywords:

parabolic trajectory, uniformly accelerated motion, ballistics

## Introduction

### Overview

A steel sphere is launched by a ballistic unit and the resulting trajectory is filmed with the aid of a video camera. The "measure Dynamics" software is used to demonstrate the dependence of the trajectory on the launching angle  $\phi$  and on the initial velocity  $u_0$ , and to determine the range and height of the trajectory. In addition, the resulting trajectory is integrated into the video, followed by a discussion of the course of the velocity.



## Equipment

Position No.	Material	Order No.	Quantity
1	Ballistic Unit	11229-10	1
2	Steel ball, d = 19 mm	02502-01	1
3	Meter scale, l = 1000 mm	03001-00	1
4	Barrel base PHYWE	02006-55	1
5	Speed measuring attachment	11229-30	1
6	Power supply 5 VDC/2.4 A	13900-99	1
7	Software "Measure Dynamics", single user	14440-61	1

## Tasks

1. Determination of the trajectory.
2. Determination of the launching angle.
3. Determination of the initial velocity.
4. Determination of the range.
5. Determination of the maximum height.
6. Integration of the trajectory into the video.
7. Integration of the velocity vectors into the video.

## Set-up and procedure

Set the ballistic unit up as shown in Figure 1 and adjust it by launching the steel sphere at a  $90^\circ$  angle, i.e. straight upwards, and by catching it in your hand. Adjust the support system so that the sphere will be launched vertically upwards. Since the launching spring of the ballistic unit has three different states of tensioning, three different initial velocities can be simulated when launching the steel sphere. Adjust a random launching angle. After the sphere has completed its trajectory, it either falls on the floor or on a table. We recommend using an empty cardboard box or similar for catching the sphere.

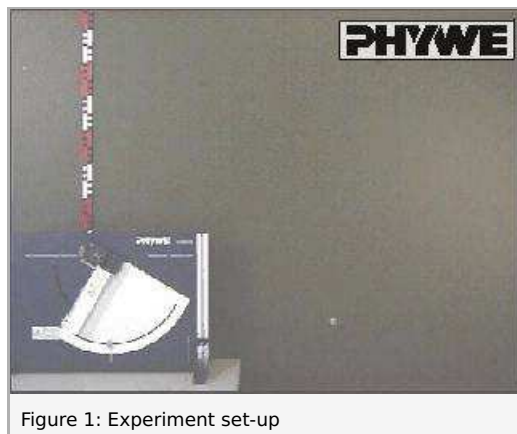


Figure 1: Experiment set-up

In terms of the video that will be recorded, the following must be taken into consideration concerning the setting and positioning of the camera:

- Set the number of frames per second to approximately 30 fps.
- Select a light-coloured, homogeneous background.
- Provide additional lighting for the experiment.
- The experiment set-up should be in the centre of the video. To ensure this, position the video camera on a tripod centrally in front of the experiment set-up.
- The experiment set-up should fill the video image as completely as possible.
- The optical axis of the camera must be parallel to the experiment set-up.
- Position a scale in the trajectory of the sphere by way of the barrel base to ensure proper scaling.

Then, the video recording process and the experiment can be started.

## Theory and evaluation

### Theory

When an object with the mass  $m$  moves in a constant gravitational field (gravitational force  $\vec{F} = m \cdot \vec{g}$ ), the trajectory lies in a plane.

If the system of coordinates is also positioned in this plane ( $x, y$  plane - see Figure 2) and if the equation of motion

$$m \frac{d^2}{dt^2} \vec{r}(t) = m \cdot \vec{g}$$

with:

$$\vec{r} = \begin{pmatrix} x \\ y \end{pmatrix}; \quad \vec{g} = \begin{pmatrix} 0 \\ -g \end{pmatrix}$$

is solved under the initial conditions

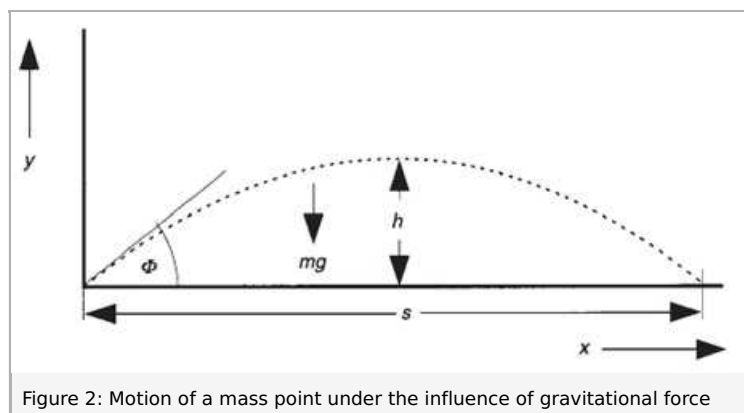
$$\vec{r}(0) = 0$$

$$\vec{v}(0) = \begin{pmatrix} v_0 \cos \phi \\ v_0 \sin \phi \end{pmatrix}$$

we obtain the coordinates  $x$  and  $y$  as a function of time  $t$ :

$$x(t) = v_0 \cos \phi t$$

$$y(t) = v_0 \sin \phi t - \frac{1}{2} g t^2$$



Based on the graph that is given above, and as a function of the angle  $\phi$ , we obtain the maximum height of the sphere from

$$h = \frac{v_0^2}{2g} (\sin \phi)^2$$

and the maximum range of the sphere from

$$s = \frac{v_0^2}{g} \cdot \sin(2\phi)$$

### Evaluation

Transfer the video that has been recorded to the computer. Then, start "measure Dynamics" and open the video under "File" - "Open video ...". Mark the start of the experiment ("Start selection" and "Time zero") and the end of the experiment ("End selection") in the video for further analysis via the menu line above the video. The experiment starts with the launch of the sphere and it ends after its first impact. Then, mark the distance that appears in the video under "Video analysis" - "Scaling ..." - "Calibration" and with the aid of the scale that is included in the experiment set-up and enter the resulting distance into the

input window. In addition, enter the frame rate that has been set for the recording process under "Change frame rate" and position the system of coordinates at the point where the sphere leaves the launching tube under "Origin and direction".

Then, the actual motion analysis can be started under "Video analysis" - "Automatic analysis" or "Manual analysis". For the automatic analysis, we recommend selecting "Motion and colour analysis" on the "Analysis" tab. Under "Options", the automatic analysis can be optimised, if necessary, e.g. by changing the sensitivity or by limiting the detection radius. Then, look for a film position in the video where the steel sphere is perfectly visible. Click the sphere. If the system recognises it, a green rectangle appears and the analysis can be started by clicking "Start".

If the automatic analysis does not lead to any satisfying results, the series of measurements can be corrected under "Manual analysis" by manually marking the object that is to be analysed.

**Task 1: Determination of the trajectory.**

In order to display the trajectory of the sphere in graphical form, select "Display" and "Diagram", click "Options", delete all of the already existing graphs, and select the graphs x (horizontal axis) - y (vertical axis). This leads to:

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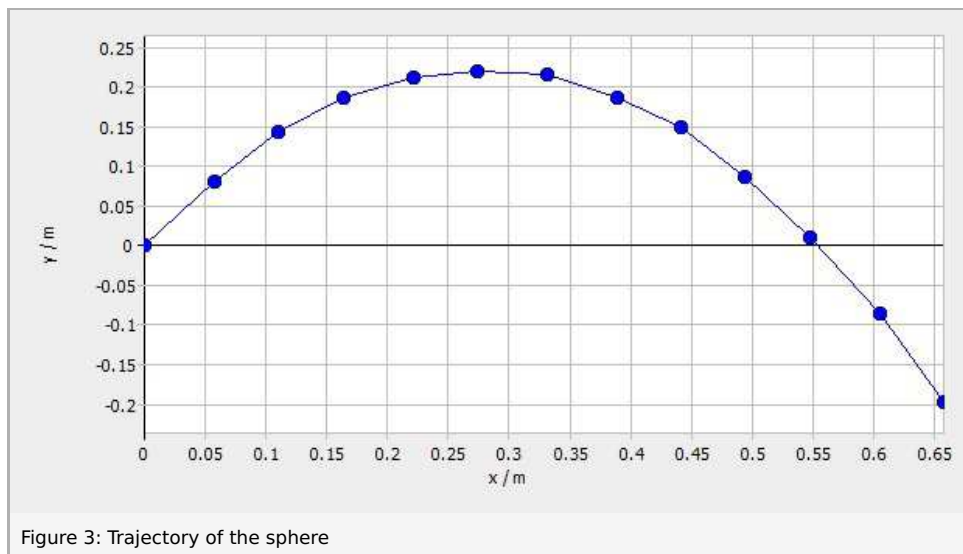
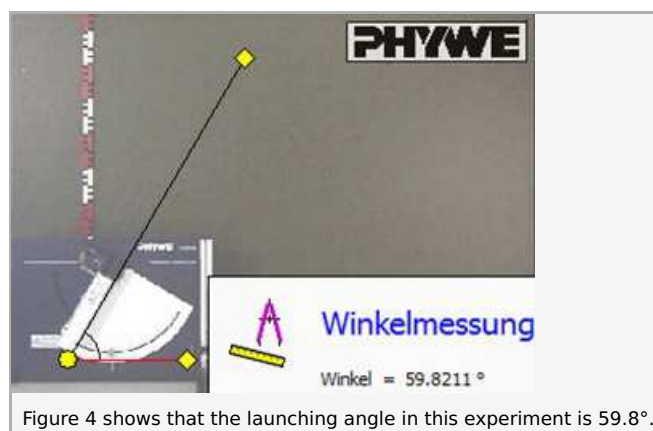


Figure 3 shows the parabolic trajectory that could have been expected from the theory. In addition, Figure 3 provides information concerning the maximum height and range of the sphere (i.e. the distance that the sphere covers until it reaches its initial height). Details concerning the values can be found in tasks 4 and 5.

**Task 2: Determination of the launching angle.**

The launching angle of the sphere can be determined either directly via the angle scale of the launching tube or by way of the angle measurement function of "measure Dynamics". In order to use the second option, open "Measure" and then "Measure angle". Position the vertex of the angle on the lower end of the launching tube. Then, position one arm of the angle parallel to the x-axis (the colour of the ray changes to red) and the other arm of the angle along the launching tube so that the angle is located in the vertex of the angle.



**Task 3: Determination of the initial velocity**

In order to determine the initial velocity, extend the worksheet by clicking "New column" in the table menu line. Then, enter "v\_total" (unit: "m/s"; formula: " $((v_x)^2 + (v_y)^2)^{0.5}$ ") into the new column. While the velocity in the x-direction remains nearly constant, i.e. the velocity loss that is caused by the air drag is neglectable at the velocities that occur during this

experiment, the velocity in the y-direction changes continuously due the gravitational acceleration. This means that the initial velocity can only be determined directly after the launch of the sphere. The initial velocity during this experiment is 2.50 m/s.

If this method is not sufficiently precise, it is also possible to determine the velocity by way of the velocity measurement attachment.

*Task 4: Determination of the range.*

The range can be determined with the aid of Figure 3 by determining the point where the sphere has reached its initial height again. In addition, this can be verified mathematically with the aid of the values of the launching angle and initial velocity from tasks 2 and 3.

The graphical method and the mathematical method provide a range of 0.55 m. This means that the theoretical equation for the range is confirmed by practical experience.

*Task 5: Determination of the maximum height.*

The maximum height can also be determined graphically with the aid of Figure 3 and also mathematically by way of the values for the launching angle and initial velocity from tasks 2 and 3 based on the theory. The height that is determined graphically is 0.22 m and the height that is determined mathematically is 0.24 m. Since the two values are very close to one another, the theoretical equation for the maximum height has been confirmed by the experiment.

*Task 6: Integration of the trajectory into the video.*

The trajectory can be integrated into the video in two different ways. One method involves the creation of a stroboscopic picture. To do so, open "Stroboscopic picture" under "Video analysis". Clicking "Start" in the dialogue box creates the following stroboscopic picture (it may be necessary to change the settings under "Options" in order to optimise the stroboscopic picture).

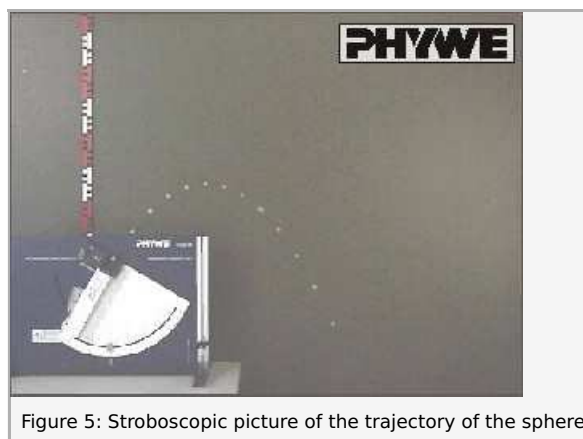


Figure 5: Stroboscopic picture of the trajectory of the sphere

The second option is to integrate the trajectory into the video. To do so, open "Filters and labels ..." under "Display", click "Add new filter", and select "Symbol". Under "Filter configuration", select the tab "Limitations" and "Filter visible". Then, select "0" as the "Start selection" and "-1" as the "End selection" under "Cutting (timeline)". Select "Line" as the icon on the "Icon" tab. Select "0" under "Trace length". This means that the symbol will remain visible in the entire video. Set the "Step" to "1". On the "Data source" tab, select the table of the sphere under "Starting point" and "0" as the "Time increment". Select "x" as the "x-coordinate". Select "y" as the "y-coordinate". Select the table of the sphere once again under "End point" and set the "Time increment" to "+1". Once again, select "x" as the "x-coordinate" and "y" as the "y-coordinate". This leads to:

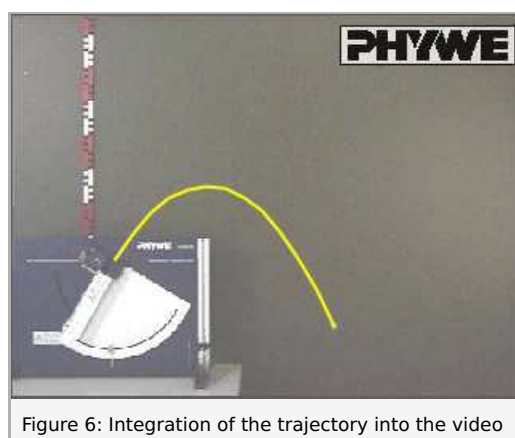


Figure 6: Integration of the trajectory into the video

*Task 7: Integration of the velocity vectors into the video.*

In the video, the development of the velocity can be demonstrated in a particularly clear manner by integrating the velocity in the x- and y-direction as well as the total velocity. To do so, open "Filters and labels ..." under "Display", click "Add new filter", and select the "Velocity arrow". Under "Filter configuration", select the tab "Limitations" and "Filter visible". Then, select "0" as the "Start selection" and "-1" as the "End selection" under "Cutting (timeline)". On the tab "Icon", select "Arrow" as the icon,

since the velocity is a vector. Select "0" under "Trace length". This means that the icon will remain visible in the entire video. Select a suitable "Step" (in this case "2") (not too few/too many vectors). On the "Data source" tab, select the table of the sphere under "Starting point" and "0" as the "Time increment". Select "x" as the "x-coordinate". Select "y" as the "y-coordinate". Select the table of the sphere once again under "End point" and set the "Time increment" to "0". In order to display the total velocity, select " $v_x$ " as the "x-coordinate" and " $v_y$ " as the "y-coordinate". In order to display the velocity in the x-direction, select " $v_x$ " as the "x-coordinate" and "Fixed value" for the "y-coordinate". Set the latter to "0". In order to display the velocity in the y-direction, select "Fixed value" as the "x-coordinate" and set it to "0", and select " $v_y$ " as the "y-coordinate". Next, choose a suitable "Stretch factor" to ensure that the vectors are neither too short nor overlapping. Then, activate "User-defined scale".

The three vectors are created with identical characteristics (step, stretch factor, ...). Ideally, different colours are selected for the three vectors under "Change icon" on the "Icon" tab. The arrows can be labelled under "Display" - "Paint ..." - "Text". This leads to:

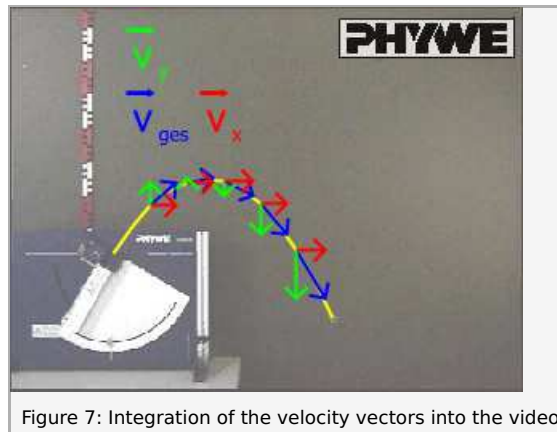


Figure 7: Integration of the velocity vectors into the video

Figure 7 shows the disintegration of the velocity vector into its x- and y-components. In addition, it becomes clear that the absolute value of the velocity in the x-direction remains nearly constant, which is indicated by the length of the associated vector. However, the velocity in the y-direction changes continuously. At the beginning, the absolute value decreases. At the maximum of the trajectory, it drops to zero. After the maximum, the direction of this velocity component is reversed. In addition, its absolute value increases. The changes of the velocity in the y-direction lead to a change of the resulting total velocity.