Resistance, phase shift and power in AC circuits

(Item No.: P2441101)

Curricular Relevance

Area of Expertise: Physics

Education Level: University

Topic: Electricity and Magnetism

Subtopic: Inductance, Electromagnetic Oscillations, AC Circuits

Experiment: Resistance, phase shift and power in AC circuits

Difficulty	Preparation Time	Execution Time	Recommended Group Size
Difficult	1 Hour	2 Hours	2 Students

Additional Requirements:

Experiment Variations:

Keywords:

impedance, phase shift, phase diagram, capacitance, self-inductance

Overview

Short description

Series circuits containing self-inductances or capacitances and ohmic resistances are investigated as a function of frequency. Measuring the electrical magnitudes with a work or power measurement instrument, real power or apparent power can be displayed directly.

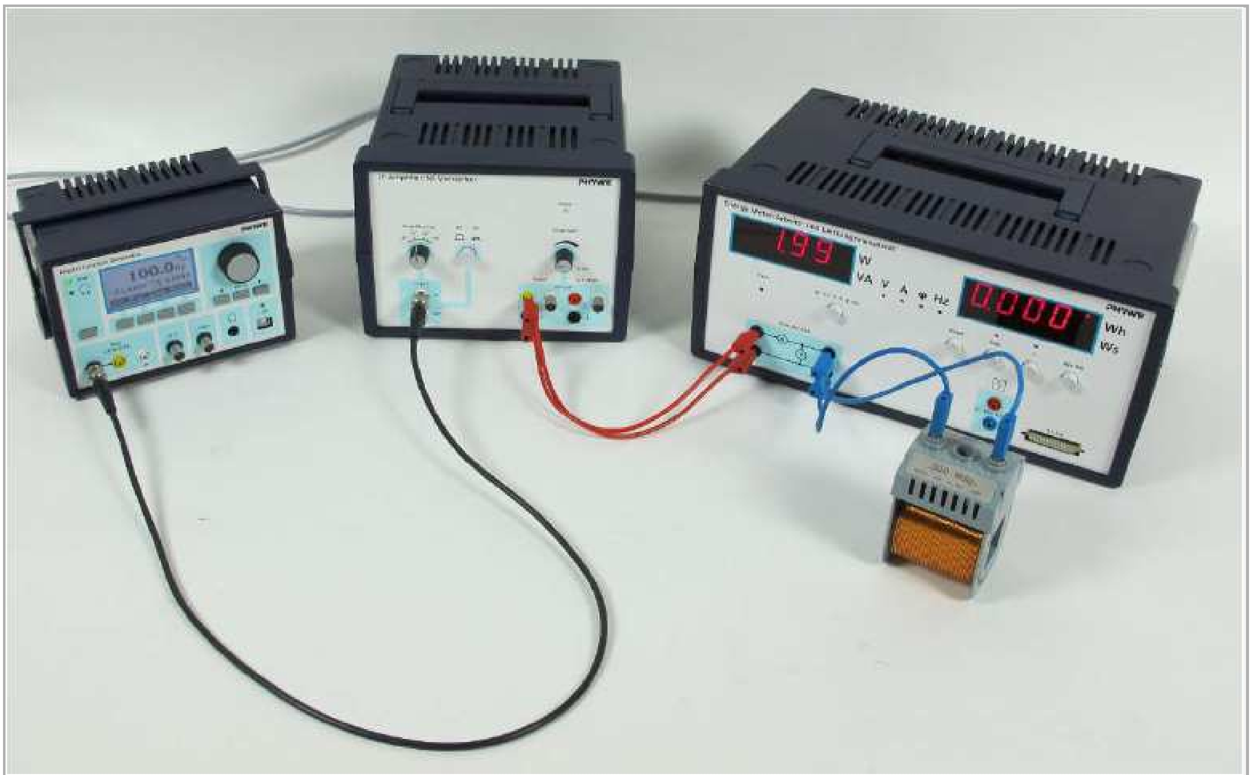

Figure 1: Experimental set-up.

Equipment

Position	Material	Order nr.	Quantity
1	Work and power meter	13715-93	1
2	LF amplifier, 220 V	13625-93	1
3	PHYWE Digital Function Generator, USB	13654-99	1
4	Coil, 300 turns	06513-01	1
5	Connection box	06030-23	1
6	Electrolytic capacitor 47 µF/63V, bipolar, G1	39105-45	1
7	Resistor 10 Ohm, 1W, G1	39104-01	1
8	Screened cable, BNC, l 250 mm	07542-10	1
9	Connecting cord, 32 A, 500 mm, black	07361-05	4

Tasks

1. Study the series circuit of self-inductance and resistor (real coil):
   a. Investigate impedance and phase shift as functions of the frequency.
   b. Investigate the relation between real power and current.
   c. Determinate self-inductance and ohmic resistance of the coil.
2. Study the series circuit of capacitor and resistor:
   a. Investigate the impedance and phase shift as functions of frequency.
   b. Investigate the relation between real power and current.
   c. Determinate capacitance and ohmic resistance of the circuit.

Set-up and Procedure

Set-up

The experimental set-up for the first task is shown in Fig. 1. Connect the frequency generator's signal output via BNC to the input of the LF amplifier. The amplified output signal is to be connected to the power meters input sockets on the left. The output sockets have to be connected with the coil (task 1) and the circuit (task 2) respectively.

Procedure

Set the signal amplitude of the digital function generator at a value between 2 V and 6 V. Don't change this setting during measurements. Set the output mode to *out*. To amplify the signal properly choose an amplification factor between 10^3 and 10^4. The measured current must be somewhat higher than 0.1 A, because this is the minimum current required by the work and power meter to determine phase shift and real power. The resistor may be loaded up to 1 W, and only for a short period of time up to 2 W.
For a detailed description of the operation of the digital function generator, the LF amplifier and the work and power meter please refer to the proper manuals.

Task 1:

Choose the output voltage at the digital frequency generator for the frequency of 200 Hz so that the current is higher than 0.1 A. The frequency is then varied within the range between 20 Hz and 200 Hz, e.g. in 20 Hz steps. Voltage, current, phase shift, real and apparent power are measured as a function of frequency. You can either change the display of the power meter manually to read the different measurands or you lock the display button. That will cause a steady change of the displayed measurands.

Task 2:

The frequency is varied within the range between 100 Hz and 1000 Hz, e. g. till 200 Hz in 20 Hz steps, and then in steps of 100 Hz or more. Voltage, current, phase shift, real and apparent power are measured as a function of frequency.

Note:

If the phase angle display changes by ± 1°, the real power display also jumps. In this case, the displayed values should be averaged.

Theory and Evaluation

Theory

To determine the impedance Z of a circuit as function of frequency the measurand pairs of U and I are used to calculate the impedance as shown in equation (1).

$Z = \frac{U}{I}$ (1)

Using a series circuit of ohmic resistance R as well as inductive and capacitive resistances X_L and X_c respectively, the impedance Z is obtained through vectorial addition of these resistances, which gives relation (2)

$Z = \sqrt{R^2 + X^2}$ (2)

With

$X_L = \omega L$

And

$X_c = \frac{1}{\omega c}$.

The phase shift ϕ between voltage and current is given through relation (3).

$\tan\phi = \frac{x}{R}$ (3)

The real power P_W in the AC circuit is calculated from voltage, current and phase shift:

$P_W = U \cdot I \cdot \cos\phi$ (4)

The angular relation

$\cos\phi = \sqrt{\frac{1}{1+\tan^2\ \phi}}$

allows for further transformation of equation (4):

$P_W = U \cdot I \cdot \sqrt{\frac{R^2}{R^2+X^2}}$

$P_W = \frac{U \cdot I \cdot R}{Z}$

$P_W = R \cdot I^2$ (5)

The apparent power P_S can simply be calculated from the voltage U and the current I :

$P_S = U \cdot I$ (6)

Evaluation

In the following the evaluation of the obtained values is described with the help of example values. Your results may vary from those presented here.

Task 1: Study the series circuit of self-inductance and resistor (real coil).
Table 1 shows the measured values for frequencies between 20 Hz and 200 Hz. For the frequency generator's signal an amplitude of 2 V had been chosen.

Figures 2 and 3 show the results for *Task 1.a*, the findings for *Task 1.b* are presented in Figure 4. All calculated values can be found in Table 2.

Task 1.c:

From equation (2) we obtain the following dependency between the impedance of the circuit Z_L and the frequency v

$$Z_L^2 = R^2 + (\omega L)^2 \cdot v^2 \qquad (7)$$

and the phase shift $\tan\phi$ between voltage and current is given by relation (8).

$$\tan\phi = \frac{\omega L}{R} \cdot V$$

From equation (7) is easily concluded that R_2 can be obtained reading the intersection with the y axis in Fig. 2:

$$R^2 = 0.857\Omega^2 \Rightarrow R = 0.926\Omega$$

The slope in Fig. 3 and equation (5) yields

$$R = 0.997\Omega$$

which gives the mean value for the ohmic resistance of the coil $R = 0.96\Omega$.

We now are able to calculate the coil's inductance from the slope of the fitted functions in Fig. 2 and 4 with equations (7) and (8) respectively:

$$(2\pi L)^2 = 0.0002H^2 \Rightarrow L = 2.25mH$$

$$\frac{2\pi \cdot L}{R} = 13.5ms \Rightarrow L = 2.06mH$$

We obtain the mean value for the inductance of the coil with $L = 2.2mH$.

Table 1: Measured values from task 1 with $U_0 = 2V$.

$\frac{V}{Hz}$	$\frac{U}{V}$	$\frac{I}{A}$	$\frac{\phi}{\circ}$	$\frac{P_w}{W}$	$\frac{P_s}{VA}$
30	1.982	1.94	12	3.760	3.845
40	2.042	1.84	18	3.603	3.759
60	2.137	1.62	31	2.970	3.463
80	2.200	1.39	42	2.272	3.058
100	2.216	1.19	49	1.731	2.637
120	2.227	1.04	55	1.338	2.315
140	2.233	0.92	59	1.055	2.054
160	2.238	0.82	63	0.835	1.839
180	2.242	0.74	66	0.676	1.663
200	2.244	0.68	68	0.568	1.516

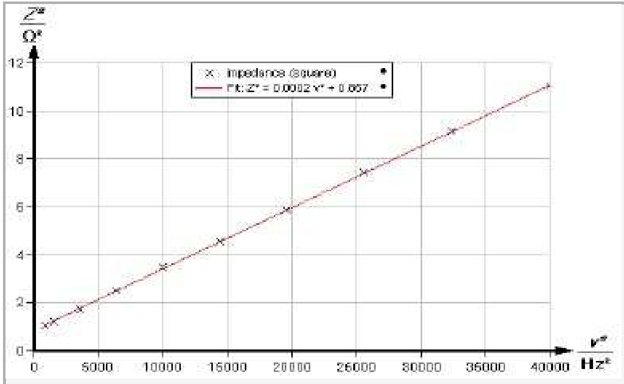

Figure 2: Self-inductance and resistor in series, Z^2 as a function of υ^2 .

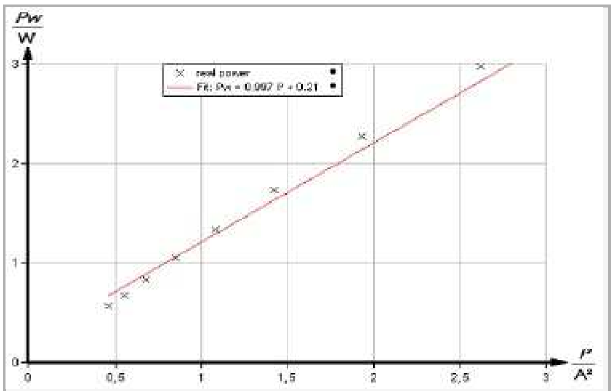

Figure 3: Self-inductance and resistor in series: P_w as a function of I_2

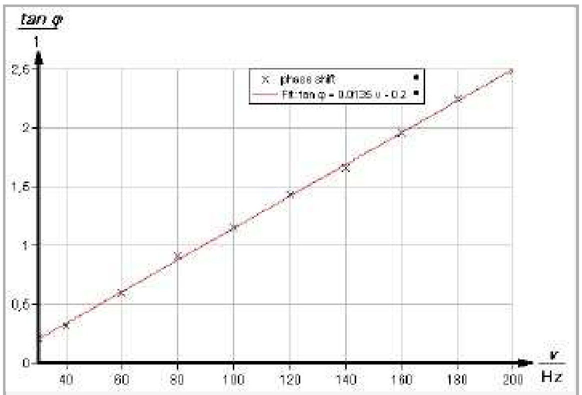

Figure 4: Self-inductance and resistor in series: $\tan\phi$ as a function of υ.

Table 3: Values calculated from the measurements of task 1. Z_L was calculated from equation (1).

$\frac{v}{Hz}$	$\frac{v^2}{Hz^2}$	$\frac{I^{-2}}{A^{2}}$	$\frac{Z_L}{\Omega}$	$\frac{Z_L^2}{\Omega^2}$	$\tan\phi$
30	900	3.76	1.02	1.04	0.21
40	1600	3.39	1.11	1.23	0.32
60	3600	2.62	1.32	1.74	0.60
80	6400	1.93	1.58	2.51	0.90
100	10000	1.42	1.86	3.47	1.15
120	14400	1.08	2.14	4.59	1.43
140	19600	0.85	2.43	5.89	1.66
160	25600	0.68	2.72	7.41	1.96
180	32400	0.55	3.02	9.13	2.25
200	40000	0.46	3.32	11.02	2.48

Task 2: Study the series circuit of capacitor and resistor.
Table 3 shows the measured values for frequencies between 100 Hz and 1000 Hz. For the frequency generator's signal an amplitude of 3 V had been chosen.
Figures 5 and 6 show the results for *Task 2.a*, the findings for *Task 2.b* are presented in Figure 7. All calculated values are shown in Table 4.

Task 2.c:

From equation (2) we obtain the following dependency between the impedance of the circuit Z_c and the frequency v with
$Z_c^2 = R^2 + \frac{1}{(2\pi \cdot C)^2} \cdot (\frac{1}{v^2})$. (9)

The phase shift between voltage and current is given with equation (10):
$\tan\phi = -(\frac{1}{2\pi \cdot C \cdot R}) \cdot (\frac{1}{v})$ (10)

From equation (9) is easily concluded that R^2 can be obtained reading the intersection with the y axis in Fig. 5:
$R^2 = 145.4\Omega^2 \Rightarrow R = 12.06\Omega$

The slope in Fig. 6 and equation (5) yields
$R = 12.0\Omega$

which results in the mean value of $R = 12.1\Omega$ for the circuit's resistance.

We now are able to calculate the capacitance from the slope of the fitted functions in Fig. 5 and 7 with equations (9) and (10) respectively:

Table 2: measured values from task 2 with $U_0 = 3V$.

$\frac{v}{Hz}$	$\frac{U}{V}$	$\frac{I}{A}$	$\frac{\phi}{\circ}$	$\frac{P_w}{W}$	$\frac{P_S}{VA}$
100	2.790	0.12	-70	0.115	0.334
120	2.789	0.13	-67	0.144	0.368
140	2.788	0.14	-64	0.175	0.980
160	2.787	0.15	-61	0.207	0.426
180	2.786	0.16	-59	0.232	0.451
200	2.785	0.17	-56	0.265	0.473
300	2.779	0.20	-45	0.395	0.558
400	2.774	0.22	-37	0.485	0.607
500	2.770	0.23	-31	0.546	0.637
1000	2.749	0.24	-17	0.634	0.662

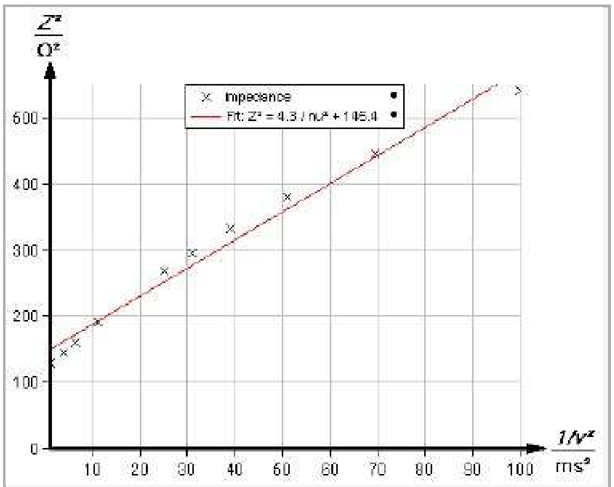

Figure 5: Capacitor and resistance in series, Z^2 as a function of $1/v^2$.

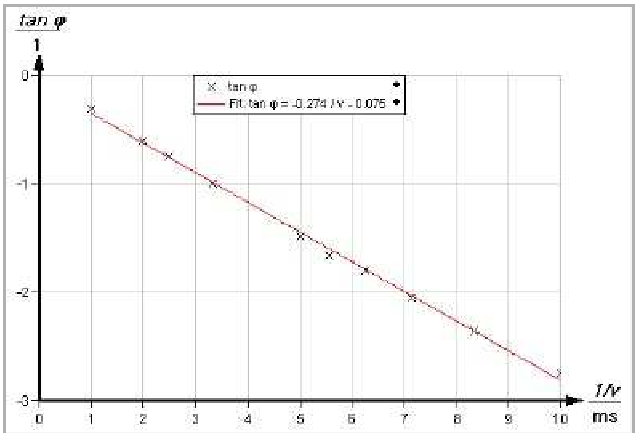

Figure 6: Capacitor and resistance in series, $\tan\phi$ as a function of $1/v$.

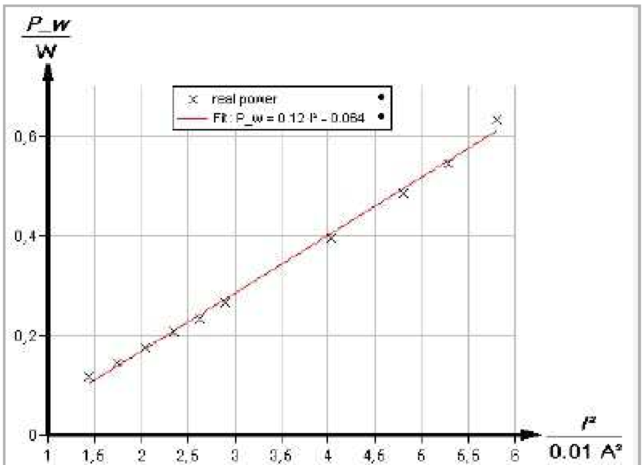

Figure 7: Capacitor and resistance in series, P_w as a function of I^2.

$$\frac{1}{(2\pi \cdot C)^2} = 4.3 \cdot 10^6 \frac{1}{F^2} \Rightarrow C = 77 \mu F$$

$$\frac{1}{2\pi \cdot R \cdot C} = 274 s \Rightarrow C = 48.4 \mu F$$

Because of the strongly scattered values in Figure 5 calculation from the observed slope gives a value for the capacitance which severely deviates from the value obtained from Figure 7. Therefore we conclude that the real capacitance must be close to $C = 48 \mu F$.

Table 4: Values calculated from the measurements of Task 2. Z_L was calculated from equation (1).

$\frac{1/v}{ms}$		$\frac{I^2}{10^{-2} A^2}$	$\frac{Z_L}{\Omega}$	$\frac{Z_L^2}{\Omega^2}$	$\tan \phi$
10.00	100.00	1.44	23.25	541	-2.75
8.33	69.44	1.74	21.13	446	-2.36
7.14	51.02	2.04	19.50	380	-2.05
6.25	39.06	2.34	18.22	332	-1.80
5.56	30.86	2.62	17.20	296	-1.66
5.00	25.00	2.89	16.38	268	-1.48
3.33	11.11	4.04	13.83	191	-1.00
2.50	6.25	4.80	12.67	160	-0.75
2.00	4.00	5.29	12.04	145	-0.60
1.00	1.00	5.81	11.41	130	-0.31