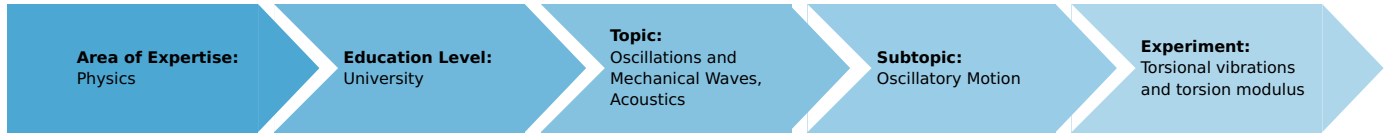


# Torsional vibrations and torsion modulus (Item No.: P2133000)

## Curricular Relevance



**Difficulty**



Difficult

**Preparation Time**



1 Hour

**Execution Time**



2 Hours

**Recommended Group Size**



2 Students

**Additional Requirements:**

**Experiment Variations:**

**Keywords:**

shear modulus, angular velocity, torque, moment of inertia, angular restoring torque, G-modulus, modulus of elasticity

## Introduction

### Overview

Bars of various materials will be exciting into torsional vibration. The relationship between the vibration period and the geometrical dimensions of the bars will be derived and the specific shear modulus for the material determined.



Fig.1: Experimental set-up for determining the torsion modulus of various bars.

## Equipment

Position No.	Material	Order No.	Quantity
1	Torsion apparatus	02421-00	1
2	Torsion rod, steel, $l = 500 \text{ mm}$ , $d = 2 \text{ mm}$	02421-01	1
3	Torsion rod, Al, $l = 500 \text{ mm}$ , $d = 2 \text{ mm}$	02421-02	1
4	Torsion rod, Al, $l = 400 \text{ mm}$ , $d = 2 \text{ mm}$	02421-03	1
5	Torsion rod, Al, $l = 300 \text{ mm}$ , $d = 2 \text{ mm}$	02421-04	1
6	Torsion rod, Al, $l = 500 \text{ mm}$ , $d = 3 \text{ mm}$	02421-05	1
7	Torsion rod, Al, $l = 500 \text{ mm}$ , $d = 4 \text{ mm}$	02421-06	1
8	Torsion rod, brass, $l = 500 \text{ mm}$ , $d = 2 \text{ mm}$	02421-07	1
9	Torsion rod, Cu, $l = 500 \text{ mm}$ , $d = 2 \text{ mm}$	02421-08	1
10	Spring Balance 1 N	03060-01	1
11	Spring balance 2,5 N	03060-02	1
12	Right angle clamp expert	02054-00	3
13	Stopwatch, digital, 1/100 s	03071-01	1
14	Sliding weight	03929-00	2
15	Support base DEMO	02007-55	1
16	Support rod, stainless steel, $l = 250 \text{ mm}$ , $d = 10 \text{ mm}$	02031-00	2
17	Support rod, stainless steel, 750 mm	02033-00	1

## Tasks

1. Static determination of the torsion modulus of a bar.
2. Determination of the moment of inertia of the rod and weights fixed to the bar, from the vibration period.
3. Determination of the dependence of the vibration period on the length and thickness of the bars.
4. Determination of the shear modulus of steel, copper, aluminium and brass.

## Set-up and procedure

The experimental set-up is arranged as shown in Fig. 1. Care should be taken that the torsion bars are perpendicular to the beam. For the static determination of the torsion modulus, the spring balance acts on the beam at  $r = 0.15$  cm. The spring balance and lever arm form a right angle. It is recommended that the steel bar, 0.5 m long, 0.002 m dia., is used for this experiment, since it is distinguished by a wide elastic range.

The steel bar is also preferable for determining the moment of inertia of the rod with the two masses arranged symmetrically (longer vibration period).

The bars must not be deflected so far that the zero-point is shifted (yield point, hysteresis) or that the bar rotates in the upper clamping sleeve. Therefore, put a right angle clamp on both ends of the torsion apparatus.

## Theory and evaluation

If a body is regarded as a continuum, and if  $\vec{r}_0$  and  $\vec{r}$  denote the position vector of a point  $p$  in the undeformed and deformed states of the body, then for small displacement vectors

$$\vec{u} = \vec{r} - \vec{r}_0 \equiv (u_1, u_2, u_3)$$

and the deformation tensor  $\hat{d}$  is:

$$d_{ik} = \frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}$$

The forces  $d\vec{F}$ , which act on a volume element of the body, the edges of the element being cut parallel to the coordinate planes, are described by the stress tensor  $\hat{\tau}$ . To each area element  $dA$ , characterised by the unit vector  $\vec{e}$  in the direction of the normal, there is assigned the stress  $\vec{p}$ :

$$\vec{p} = \frac{d\vec{F}}{dA}$$

$$\vec{p} = \vec{e} \cdot \hat{\tau}$$

Hooke's law provides the relationship between  $\hat{d}$  and  $\hat{\tau}$ :

$$\tau_{ik} = \sum_{l,m} c_{ik}^{lm} d_{lm}$$

The tensor  $\hat{c}$  is symmetrical for an elastic body, so that only 21 of the 81 components remain. For isotropic elastic bodies, this number is further reduced to 2 values, namely the modulus of elasticity  $E$  and the shear modulus  $G$  or the Poisson's ratio  $\mu$ :

$$\tau_{11} = \frac{E}{1 + \mu}$$

$$\left\{ d_{11} + \frac{\mu}{1 - 2\mu} (d_{11} + d_{22} + d_{33}) \right\}$$

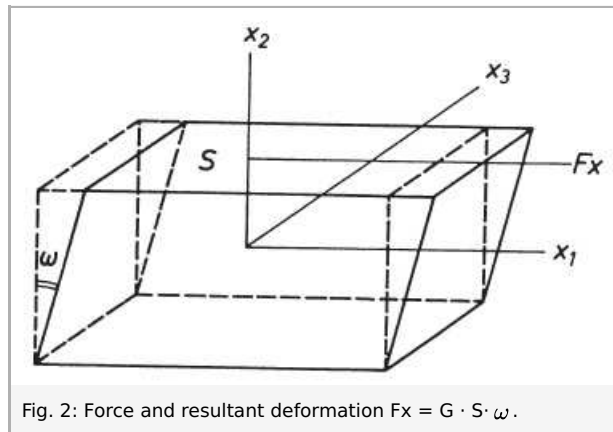


Fig. 2: Force and resultant deformation  $F_x = G \cdot S \cdot \omega$ .

$$\tau_{12} = G d_{12} = \frac{1}{2} \frac{E}{1 + \mu} d_{12} \quad (1)$$

and analogously for  $\tau_{22}, \tau_{33}, \tau_{13}, \tau_{23}$ .

Using the notations of Figs. 2 and 3, equation (1) can be written:

$$dF_x = g \cdot \omega \cdot dS = G \cdot \frac{r\phi}{L} \cdot r dr \cdot d\alpha.$$

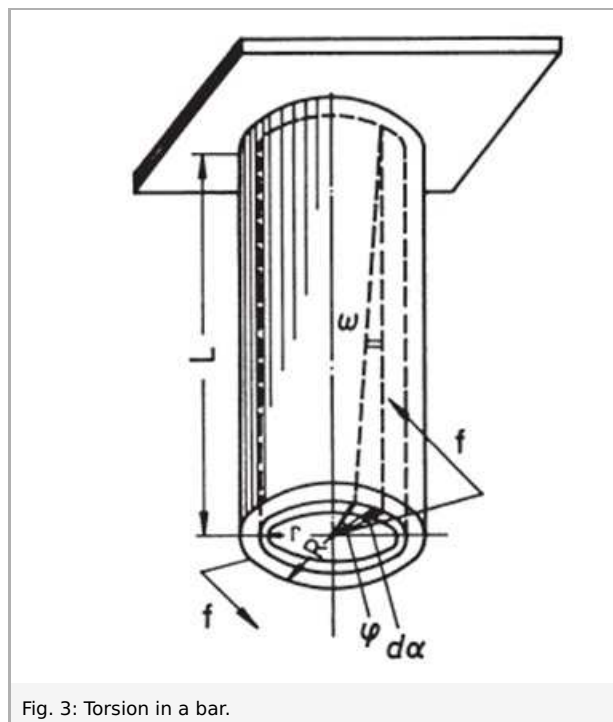


Fig. 3: Torsion in a bar.

From this, the total torque a ring is obtained

$$dT_Z = \int_0^{2\pi} dF_x \cdot r = \frac{2\pi G r^3 \phi dr}{L}$$

and from this the total torque

$$T_Z = \int_0^R dT_Z = \frac{\pi}{2} \cdot G \cdot \phi \frac{R^4}{L}.$$

From the definition of the angular restoring torque or torsion modulus  $D_T$ ,

$$T_Z = D_T \cdot \phi \tag{2}$$

one obtains

$$D_T = \frac{\pi}{2} \cdot \frac{R^4}{L} \cdot G.$$

From Newton's basic equation for rotary motion

$$\vec{T} = \frac{d}{dt} \vec{L},$$

where the angular momentum  $\vec{L}$  is related to the angular velocity  $\vec{\omega}$  and the inertia tensor  $\hat{I}$  in accordance with

$$\vec{L} = \hat{I} \cdot \vec{\omega}$$

and from (2), one obtains

$$\frac{d^2\phi}{dt^2} + \frac{D_T}{I_Z} \phi = 0.$$

The period of this vibration is

$$T = 2\pi \sqrt{\frac{I_Z}{D_T}} \tag{3}$$

or

$$T = \frac{2\pi}{R^2} \sqrt{I_Z \cdot \frac{2}{\pi} \cdot \frac{L}{G}} \tag{4}$$

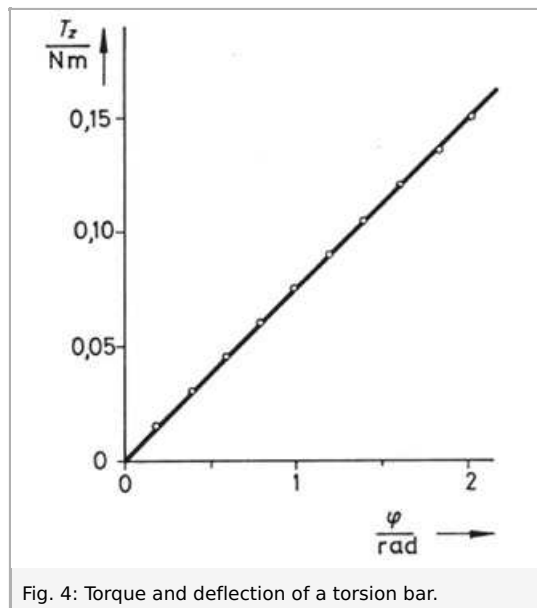
In Task 1,  $D_T$  is determined in accordance with (2).

From the regression line to the measured values of Fig. 4 with the linear statement  $Y = A + BX$ ,

the slope

$B = 0.238 \text{ Nm/rad}$  (see(2))

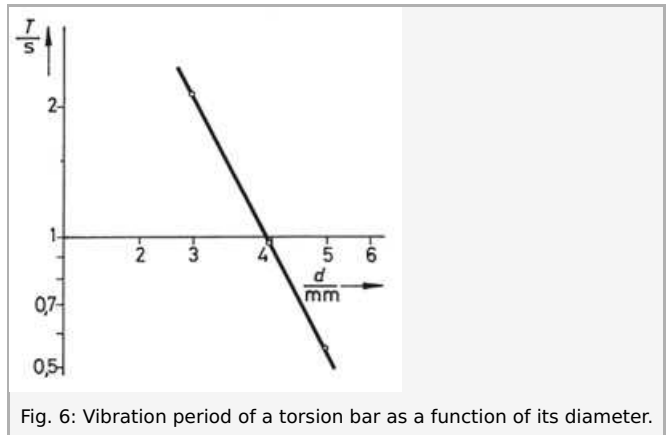
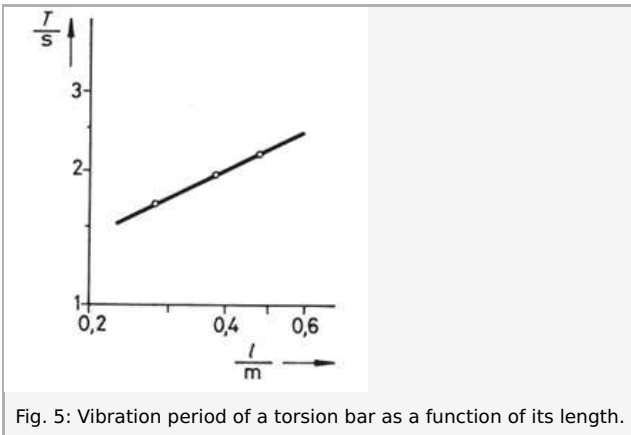
is obtained.



From (3), the moment of inertia of the vibrating bars and masses is determined for the same torsion bar:

$$I_Z = 9.24 \cdot 10^{-3} \text{ kg m}^2.$$

Figures 5 and 6 show the relationship between the vibration period and the length and diameter of the aluminium bars.



From the regression line to the measured values of Fig. 5 with the exponential statement  $Y = A \cdot X^B$

the exponent

$$B = 0.49 \pm 0.01 \text{ (see(4))}$$

is obtained.

From the regression line to the measured values of Fig. 6 with the exponential statement

$$Y = A \cdot X^B$$

the exponent

$$B = -1.99 \pm 0.01 \text{ (see(4))}$$

is obtained.

Finally, the shear modulus  $G$  is determined from (4) for Cu, Al, steel and brass.

$$G_{Cu} = 38 \cdot 10^9 \text{ N/m}^2 \text{ rad}$$

$$G_{Al} = 24 \cdot 10^9 \text{ N/m}^2 \text{ rad}$$

$$G_{Steel} = 76 \cdot 10^9 \text{ N/m}^2 \text{ rad}$$

$$G_{Brass} = 32 \cdot 10^9 \text{ N/m}^2 \text{ rad}$$