Wheatstone bridge (Item No.: P2410200)

Curricular Relevance

Area of Expertise: Physics
Education Level: University
Topic: Electricity and Magnetism
Subtopic: Electric Current and Resistance
Experiment: Wheatstone bridge

Difficulty	Preparation Time	Execution Time	Recommended Group Size
Intermediate	1 Hour	2 Hours	2 Students

Additional Requirements:

- Adhesive tape, opaque

Experiment Variations:

- With the resistance board (item no. 06108-00), another task to determine the conductivity of thin wires can be performed

Keywords:

Kirchhoff's law, Conductor, Circuit, Voltage, Resistance, Resistivity, Parallel connection, Series connection

Overview

Short description

Principle

The Wheatstone bridge is used to determine an unknown resistance with high accuracy by adjusting a connected combination of known resistors.

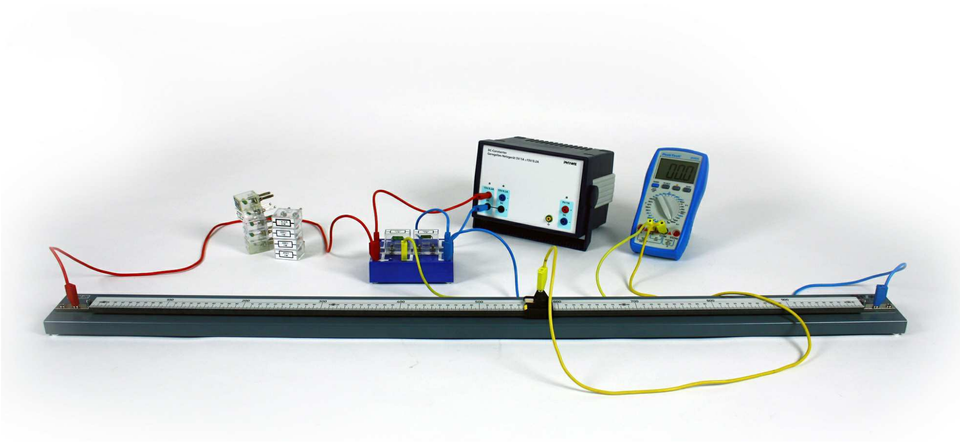

Fig. 1: Experimental set-up of the Wheatstone bridge.

Equipment

Position No.	Material	Order No.	Quantity
1	PHYWE power supply, 230 V, DC: 0...12 V, 2 A / AC: 6 V, 12 V, 5 A	13506-93	1
2	Digital multimeter 2005	07129-00	1
3	Slide wire meas. bridge, simple	07182-00	1
5	Connection box	06030-23	1
6	Resistor 1 Ohm 2%, 2W, G1	06055-10	1
7	Resistor 2 Ohm 2%, 2W, G1	06055-20	1
8	Resistor 5 Ohm 5%, 2W, G1	06055-50	1
9	Resistor 10 Ohm, 1W, G1	39104-01	1
10	Resistor 47 Ohm, 1W, G1	39104-62	1
11	Resistor 100 Ohm, 1W, G1	39104-63	1
12	Resistor 150 Ohm, 1W, G1	39104-10	1
13	Resistor 220 Ohm, 1W, G1	39104-64	1
14	Resistor 330 Ohm, 1W, G1	39104-13	1
15	Resistor 680 Ohm, 1W, G1	39104-17	1
16	Connecting cord, 32 A, 1000 mm, red	07363-01	1
17	Connecting cord, 32 A, 500 mm, red	07361-01	1
18	Connecting cord, 32 A, 1000 mm, yellow	07363-02	2
20	Connecting cord, 32 A, 1000 mm, blue	07363-04	1
21	Connecting cord, 32 A, 500 mm, blue	07361-04	1
Additional materials			
	Adhesive tape, opaque		

Optional equipment for task 4 to determine small resistances of thin wires (NOT included in the supply):

Position No.	Material	Order No.	Quantity
	Resistance board, metal	06108-00	1
	Connecting cord, 32 A, 1000 mm, red	07363-01	1
	Connecting cord, 32 A, 500 mm, yellow	07361-02	1

Tasks

1. Determination of unknown resistors with the Wheatstone bridge.
2. Determination of the total resistance of resistors in series.
3. Determination of the total resistance of resistors in parallel.
4. **Optional (see required equipment):** Measurement of low resistance and determination of the electrical resistivity of CuNi (Constantan).

Set-up and procedure

Theory

The Wheatstone bridge consists of four resistors that are connected as shown in Fig. 2. A voltage source is connected to the junctions **a** and **c,** whilst the ammeter ***G*** measures the current flow between the junctions **b** and **d**.

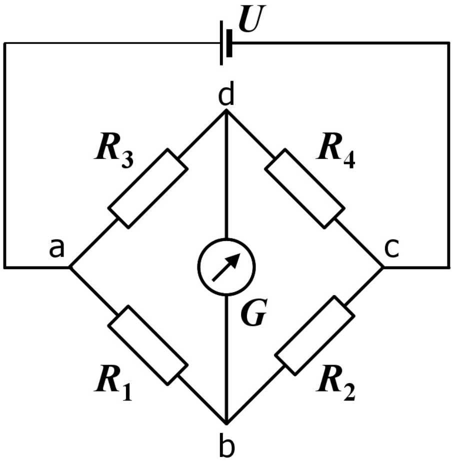

Fig. 2: Wheatstone bridge, basic circuit diagram.

Kirchhoff's second law implies that the application of a certain voltage between **a** and **c** causes an equal potential drop across the legs **a–b–c** and **a–d–c**.

Both legs serve as a particular voltage divider. The potentials at the junctions **b** and **d** depend on the proportions of the resistors along the respective legs

$$V_\mathrm{b} = \frac{R_2}{R_1+R_2} \cdot V_\mathrm{source} \text{ and } V_\mathrm{d} = \frac{R_4}{R_3+R_4} \cdot V_\mathrm{source}. \quad (1)$$

If the resistor proportions are adjusted in a way so that the potentials V_b and V_d equalize, the current flow through the ammeter vanishes. The detection of zero current is very easy and could also be carried out with a simple galvanometer. This state is called the **point of balance** of the Wheatstone bridge. In that case, Eq. 1 can be combined:

$$\frac{R_2}{R_1+R_2} = \frac{R_4}{R_3+R_4} \quad (2)$$

$$\Leftrightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4} \quad (3)$$

If three resistors are known and the fourth one, e.g. R_3, should be identified, it can be calculated after setting the bridge to the point of balance

$$R_3 = \frac{R_1}{R_2} \cdot R_4. \quad (4)$$

In the present experiment, the resistance in the lower leg is replaced by a slide wire potentiometer as shown in Fig. 3, where a slider contact can be moved along the length of a wire to divide its total resistance in two distinct parts. The wire is made of a homogenous material with a uniform diameter, therefore, its resistance can be specified by

$$R = \rho \cdot \frac{l}{A} \quad (5)$$

where ρ denotes the electrical resistivity of the wire material, l is the length of the wire, and A is its cross-section. Hence, the resistance of the wire depends on geometrical as well as material-specific properties.

Uniform resistivity and cross-section given, the resistance increases proportionally with the length. This implies that the slide wire resistance ratio R_1/R_2 can be expressed by its lengths proportion l_1/l_2.

Set-up

In advance, up to five randomly selected resistors must be prepared for the measurement. To hide the resistance information, the printed values and optionally the transparent case of the chosen resistors have to be covered with adhesive tape. For separation during the measurement and verification afterwards, the resistors should be labeled with R_{x1} ... R_{x5}. In the following measuring examples, the used designation is:

R_{x1}	R_{x2}	R_{x3}	R_{x4}	R_{x5}
220 Ω	2 Ω	47 Ω	5 Ω	680 Ω

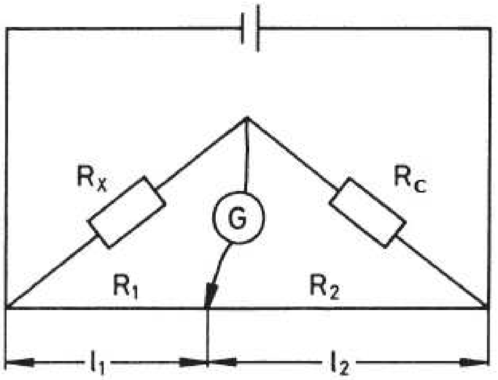

Fig. 3: Wheatstone bridge circuit diagram.

The experimental set-up is shown in Fig. 1. According to Fig. 4, the first unknown resistance R_x and a known resistor R_C (e.g. 100 Ω) are placed on the connection box. The short red cord connects R_x with the red connector of the DC output, while the long red cord connects this junction with the left connector of the slide wire measuring bridge. Likewise, the blue cords are connected with the comparison resistor R_C, the blue connector of the DC output, and the right connector of the slide wire. In between, the yellow cords connect the ammeter ***G*** (mA scale, DC mode) with the junction between R_x and R_C as well as the slider of the measuring bridge. The circuit diagram is illustrated in Fig. 3.

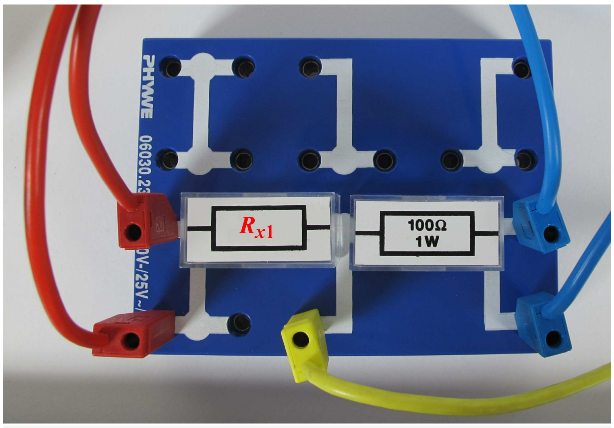

Fig. 4: Set-up of the upper leg of the Wheatstone bridge with an unknown resistance.

Procedure

1. Determination of unknown resistors

- To obtain the unknown resistance R_x, the slider of the measure bridge has to be moved in a position so that the ammeter displays zero current.
- Depending on the chosen resistors, it is possible that you will detect a remaining current in every slider position. In that case, you have to replace the comparison resistor R_C with a different one. The same procedure is advised if the slider position is close to the wire ends, because the measurement precision will otherwise decrease.
- Determine the slider position l_1 on the ruler and the associated position l_2 = 100 cm − l_1.
- Replace R_x and repeat the procedure until all five unknown resistors are determined.

Note:
The power supply provides a current of up to 2 A at a voltage of 12 V. If the current should exceed that threshold due to e.g. a small resistance in the circuit, the device will reduce the voltage to prevent damage. This is indicated by the glowing of the red LED above the current selector. Nevertheless, the measurement and the results will not be affected by that short-circuit protection.

2. Determination of the total resistance of resistors in series.

- To determine the resistance of two or three unknown resistors connected in series, the upper leg of the Wheatstone bridge has to be altered according to Fig 5 or Fig. 6, respectively. Resistance R_x is replaced by different combinations of the unknown resistors, while one of the known resistors remains as R_C.
- Perform the measurements just as you did in task 1 for at least two combinations of two, and two combinations of three resistors. Determine the corresponding slider positions l_1 (as well as l_2).

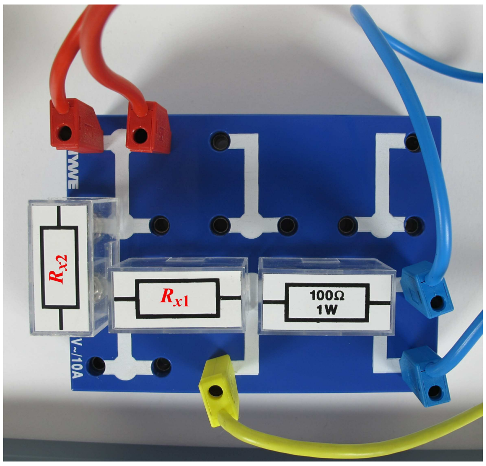

Fig. 5: Set-up for two unknown resistors connected in series.

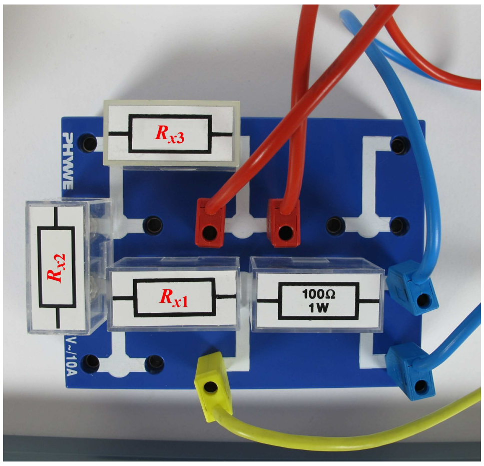

Fig. 6: Set-up for three unknown resistors connected in series.

3. Determination of the total resistance of resistors in parallel.

- To study the effect of two resistors connected in parallel, the set-up has to be adjusted as shown in Fig. 7. Resistance R_x is substituted by two unknown resistors in a parallel circuit, while one of the known resistors serves as R_C.
- Repeat the steps of task 1 for at least three different combinations of resistors and determine the corresponding slider positions l_1 (as well as l_2), again.

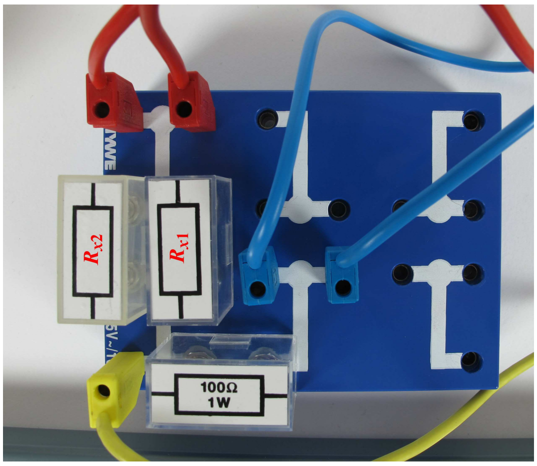

Fig. 7: Set-up of the upper leg of the Wheatstone bridge with two unknown resistors connected in parallel.

4. ***Optional:*** *Determination of the electrical resistivity of CuNi.*

Note: For this experimental part, additional equipment is needed.

- One of the benefits of the Wheatstone bridge is the possibility to precisely determine a low resistance. For demonstration, the resistance of several Constantan wires with varying diameters (and identical length l = 100 cm) should be measured. Thereof, the electrical resistivity of Constantan can be obtained.

- For this purpose, R_C should be replaced by a resistor of approximately 1 Ω, and the CuNi44 wire No. 1 of the resistance board has to be connected as R_x (see Fig. 8 and 9).
- Again, the measurement has to be carried out like before until the bridge is balanced.
- Register the slider position l_1 (as well as l_2) and repeat the measurement for the remaining CuNi44 wires No. 2, 3 and 5.

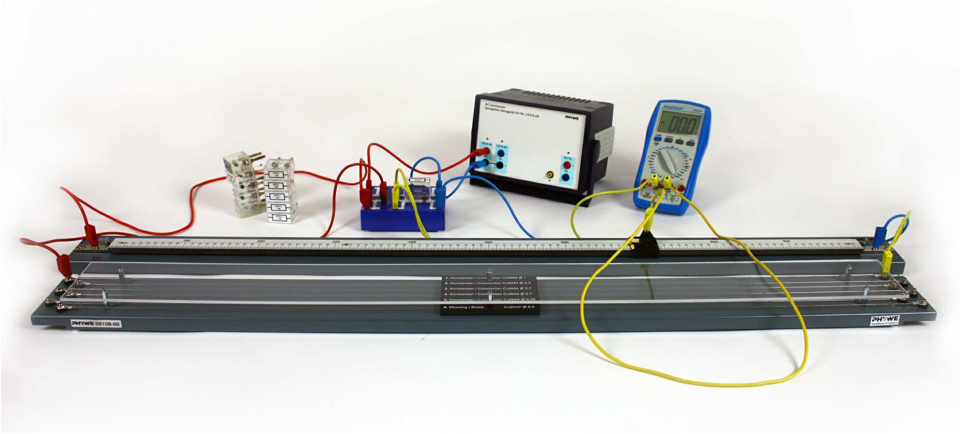

Fig. 8: Experimental set-up using the resistance board.

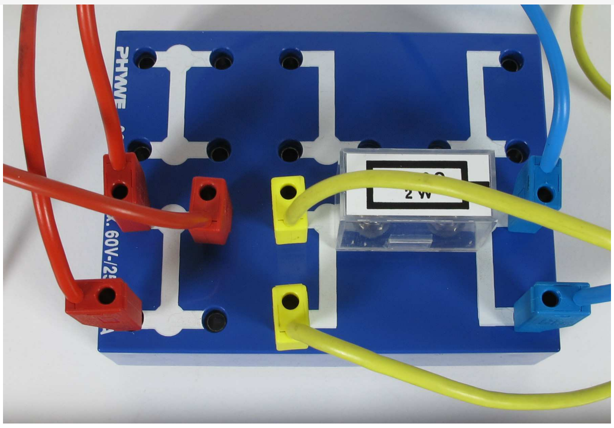

Fig. 9: Set-up of the connection box for use of the resistance board with the Wheatstone bridge.

Evaluation

Kirchhoff's first law implies that the sum over all currents directed towards or away from a junction has to be zero. At the point of balance, with no current flowing through the ammeter, the current flow must remain conserved along both legs of the Wheatstone bridge. Thus Eq. 4 and 5 give the unknown resistance

$$R_x = \frac{l_1}{l_2} \cdot R_\mathrm{C}\,. \tag{6}$$

1. Determination of unknown resistors

Calculate the resistance of the unknown resistors using Eq. 6 and the measured lengths on the slide wire. Reveal the indicated values of the used resistors and compare them to the calculated results (see example in Tab. 1).

You will notice that the deviation increases for values measured close to the wire ends. Nevertheless, the absolute values of measured and actual resistance are in good agreement. In order to reduce tolerance, exchange the comparison resistor R_C with a resistor of comparable magnitude to R_x.

Table 1: Evaluation of unknown resistors with the Wheatstone bridge.

Labelling	R_C in Ω	l_1 in mm	l_2 in mm	R_x in Ω	actual R_x in Ω	deviation in %
R_{x1}	100	690	310	223	220	1.4
R_{x2}	100	22	978	2.25	2	12.5
R_{x3}	100	317	683	46.4	47	−1.3
R_{x4}	100	51	949	5.4	5	8.0
R_{x5}	100	875	125	700	680	2.9

2. *Determination of the total resistance of resistors in series.*

Calculate the total resistance R_x of the unknown resistors in the series circuit using Eq. 6 and the measured lengths on the slide wire. Reveal the indicated values of the used resistors and compare them to the calculated results (see example in Tab. 2).

Table 2: Evaluation of resistors in a series circuit.

Used resistors	R_C in Ω	l_1 in mm	l_2 in mm	R_x in Ω	actual R_x in Ω*	deviation in %
R_{x3}, R_{x5}	100	882	118	747.5	47 + 680 = 727	2.8
R_{x1}, R_{x4}	100	690	310	222.6	220 + 5 = 225	−1.1
R_{x1}, R_{x2}, R_{x3}	100	732	268	273.1	220 + 2 + 47 = 269	1.5
R_{x1}, R_{x3}, R_{x5}	100	909	91	998.9	220 + 47 + 680 = 947	5.5

* calculation according to Eq. 7

According to Kirchhoff's second law, the voltage drop across each leg of the Wheatstone bridge sums up to the total applied voltage at the point of balance. Because of the constant current flow along each leg, Ohm's law gives a total resistance that is equal to the sum of each individual resistance value in a series circuit

$$R_{\text{tot}} = \sum_{i=1}^{n} R_i. \tag{7}$$

3. *Determination of the total resistance of resistors in parallel.*

Calculate the total resistance R_x of the unknown resistors in the parallel circuit using Eq. 6 and the measured lengths on the slide wire. Reveal the indicated values of the used resistors and compare them to the calculated results (see example in Tab. 3).

Table 3: Evaluation of resistors in a parallel circuit.

Used resistors	R_C in Ω	l_1 in mm	l_2 in mm	R_x in Ω	actual R_x in Ω**	deviation in %
R_{x1}, R_{x3}	100	282	718	39.3	220 and 47 ⇒ 38.7	1.4
R_{x2}, R_{x3}	100	17	983	1.7	2 and 47 ⇒ 1.9	−9.9
R_{x4}, R_{x5}	100	44	956	4.6	5 and 680 ⇒ 5.0	−7.3

** calculation according to Eq. 8

According to Kirchhoff's first law, the total current in a parallel circuit of resistors equals the sum over the currents in each individual branch whereas the voltage is the same for each component. Thus the total resistance of a parallel circuit can be determined using Ohm's law and adding up the reciprocal values of each components' resistance:

$$\frac{1}{R_{\text{tot}}} = \sum_{i=1}^{n} \frac{1}{R_i} \, . \tag{8}$$

Adding a parallel branch to the circuit increases the current flow because the present resistance can be bypassed along the extra branch. Therefore, the total resistance of a parallel circuit is always lower than the resistance value of the smallest particular resistor (see Tab. 3).

*4. **Optional:** Determination of the electrical resistivity of CuNi.*

Calculate the total resistance R_x of the wires in the parallel circuit using Eq. 6 and the measured lengths on the slide wire (see example in Tab. 4).

Table 4: Resistance of various Constantan wires with length l = 1 m.

diameter d in mm	l_1 in mm	l_2 in mm	R in Ω
1	392	608	0.645
0.5	717	283	2.53
0.7	564	436	1.29
0.35	840	160	5.25

Corresponding to Eq. 5, the electrical resistivity is an intrinsic property that can be calculated from the measured resistance of the wire and its geometry

$$\rho = R \cdot \frac{A}{l} \, . \tag{9}$$

Given that the resistance has been obtained for several wires made of the same material, the electrical resistivity of CuNi can be determined with increased accuracy by plotting the resistance R against l/A. The proportionality factor ρ corresponds to the slope of the linear fit (as shown in Fig. 10)

$$R = \rho \frac{l}{A} = \rho \frac{l}{\pi r^2} = \rho \frac{4 \cdot l}{\pi d^2} \, . \tag{10}$$

The slope of the fit in Fig. 10 gives an electrical resistivity of

$$\rho = 5.06 \cdot 10^{-7} \, \Omega \cdot \text{m} \tag{11}$$

that is in very good agreement with the literature value for Constantan of approximately $5.0 \cdot 10^{-7}$ Ω·m.

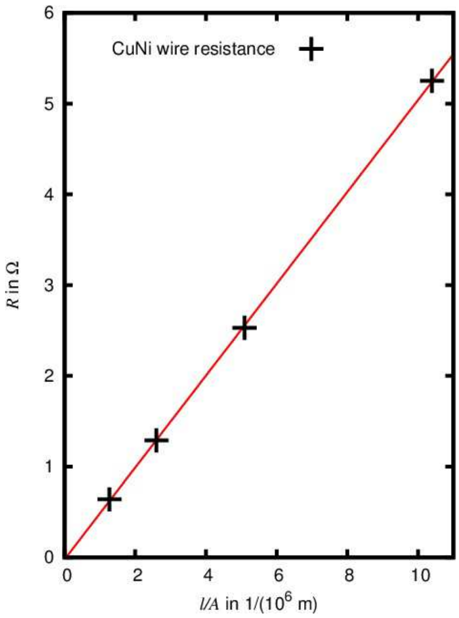

Fig. 10: Resistance of a conductor wire as a function of its geometry l/A.

Remark

- In this experiment, the resistance values of the four resistors have been taken into account, only. Actually, the resistance of the used cords, the connections as well as the ammeter could not be avoided and have been neglected as there is only a minor influence on the results.
- It is possible to determine very small resistances with the Wheatstone bridge gaining a high accuracy. Yet, the results lose precision when moving the slider to the far end positions of the potentiometer. For best results, the slider should remain close to the central position, and instead, the comparison resistor should be replaced until its magnitude matches that of the unknown resistor.