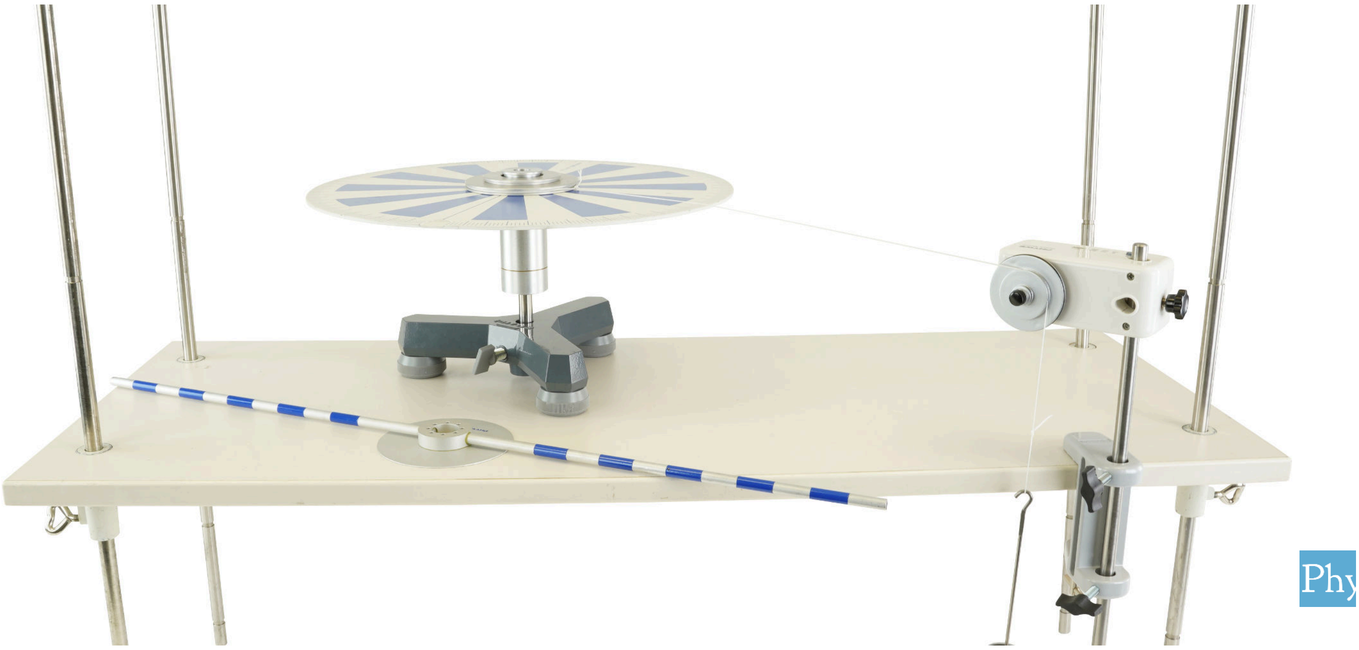
Moment of inertia and angular acceleration with Cobra SMARTsense and a precision pivot bearing



Physics	Mechanics	Energy conservation & impulse

Difficulty level	Group size	Preparation time	Execution time
hard	2	10 minutes	20 minutes

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General information

Application

Fig.1: Experimental set-up

The moment of inertia and the angular acceleration are fundamental for the field of mechanics. As such it's understanding is very important for the study of this field.

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Other information (1/2)

Prior knowledge

The prior knowledge for this experiment is found in the Theory section.

Main principle

A known torque is applied to a body that can rotate about a fixed axis with minimal friction. Angle and angular velocity are measured over the time and the moment of inertia is determined. The torque is exerted by a string on a wheel of known radius with the force on the string resulting from the known force of a mass in the earth's gravitational field. The known energy gain of the lowering mass is converted to rotational energy of the body under observation.

Other information (2/2)

Learning objective

The goal of this experiment is to investigate the moment of inertia..

Tasks

1. Measure the angle of rotation vs. time for a disc. Apply several different constant torques, generated with various forces on three different radii. Calculate the moment of inertia of the disc.
2. Measure the angle of rotation vs. time for a bar, with two masses mounted at different distances to the axis of rotation. Calculate the moment of inertia of the bar.
3. Calculate the rotational energy and the angular momentum of the disc over the time. Calculate the energy loss of the weight from the height loss over the time and compare.

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Theory (1/7)

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The angular momentum $\vec{J}$ of a single particle at position $\vec{r}$ with velocity $\vec{v}$, mass m and momentum $\vec{p} = m\vec{v}$ is defined as

$$\vec{J} = \vec{r} \times \vec{p}$$

and the torque $\vec{T}$ from the force $\vec{F}$ is defined as

$$\vec{t} = \vec{r} \times \vec{F}$$

With torque and angular momentum depending on the origin of the reference frame. The change of $\vec{J}$ in time is

$$\frac{\mathrm{d}\vec{J}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(\vec{r} \times \vec{p}) = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$$

Theory (2/7)

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and with $\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} \times \vec{p} = \vec{v} \times m\vec{v} = 0$

and Newton's law $\vec{F} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$ the equation of movement becomes

$$\vec{T} = \frac{\mathrm{d}\vec{J}}{\mathrm{d}t} \quad (1)$$

For a system of N particles with center of mass $\vec{R}_{c,m}$ and total linear momentum $\vec{P} = \sum m_i \vec{v}_i$ the angular momentum is

$$\vec{J} = \sum_{i=1}^{N} m_i \left(\vec{r}_i - \vec{R}_{\text{c.m.}}\right) \times \vec{v}_i + \sum_{i=0}^{N} m_i \vec{R}_{\text{c.m.}} \times \vec{v}_i = \vec{J}_{\text{c.m.}} + \vec{R}_{\text{c.m.}} \times \vec{P}$$

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Theory (3/7)

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Now the movement of the center of mass is neglected, the origin set to the center of mass and a rigid body assumed with $\vec{r}_i - \vec{r}_j$ fixed. The velocity of particle i may be written as $\vec{v}_i = \vec{\omega} \times \vec{r}_i$ with vector of rotation

$\vec{\omega} = \frac{\mathrm{d}\vec{\varphi}}{\mathrm{d}t}$ (2)

constant throughout the body. Then

$\vec{J} = \sum m_i \vec{r}_i \times \vec{v}_i = \sum m_i \vec{r}_i \times (\vec{\omega} \times \vec{r}_i)$

with $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \vec{b}\,(\vec{a} \cdot \vec{c}) - \vec{c}\left(\vec{a} \cdot \vec{b}\right)$

is $\vec{J} = \sum m_i(\vec{\omega} \cdot r_i^2 - \vec{r}_i(\vec{r}_i \cdot \vec{\omega}))$

Theory (4/7)

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with $\vec{r}_i \cdot \vec{\omega} = x_i + \omega_x + y_i + \omega_y + z_i + \omega_z$

and $J_z = \omega_z \sum m_i(r_i^2 - z_i^2) - \omega_z \sum m_i z_i x_{-i} - \omega_y \sum m_i z_i y_i$

The inertial coefficients or moments of inertia are defined as

$I_{x,x} = \sum m_i(r_i^2 - x_i^2)$ (3.1)

$I_{x,y} = -\sum m_i x_i y_i$ (3.2)

$I_{x,z} = -\sum m_i x_i z_i$ (3.3)

and with the matrix $\hat{I} = \{I_{k,l}\}$ it is

$\vec{J} = \hat{I} \cdot \vec{\omega}$ (4)

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Theory (5/7)

and for the rotational acceleration $\vec{\alpha} = \frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t} = \frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}$, it follows

$$\vec{T} = \frac{\mathrm{d}\vec{J}}{\mathrm{d}t} = \hat{I} \cdot \frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t} = \hat{I} \cdot \vec{\alpha}$$

The rotational energy is

$$E = \frac{1}{2}\sum m_i v_i^2 = \frac{1}{2}\sum m_i(\vec{\omega} \times \vec{r}_i)^2 = \frac{1}{2} I_{k,l}\omega_k\omega_l$$

Sum convention: sum up over same indices using

$$\left(\vec{a} \times \vec{b}\right)\left(\vec{c} \times \vec{d}\right) = (\vec{a} \cdot \vec{c})\left(\vec{b} \cdot \vec{d}\right) - \left(\vec{a} \cdot \vec{d}\right)\left(\vec{b} \cdot \vec{c}\right)$$

The coordinate axes can always be set to the "principal axes of inertia" so that none but the diagonal elements of the matrix $I_{k,k} \neq 0$, In this experiment only a rotation around the z-axis can occur and $\vec{\omega} = \hat{e}_z\omega_z = \hat{e}_z\omega$ with the unit vector $\hat{e}_z$. Then, the energy is

$$E = \frac{1}{2} I_{z,z}\omega^2 \quad (5)$$

Theory (6/7)

The torque $T = m_\mathrm{a}(g - a)r_\mathrm{a}$ is nearly constant in time since the acceleration $a = \alpha \cdot r_\mathrm{a}$ of the mass m_a used for accelerating the rotation is small compared to the gravitational acceleration $g = 9.81\,\mathrm{m/s}^2$ and the thread is always tangential to the wheel with R_a:

$$T = I_{z,z}\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} = m_\mathrm{a} g r_\mathrm{a} \quad (6)$$

$$\omega(t) = \omega(t=0) + \frac{m_\mathrm{a} g r_\mathrm{a}}{I_{z,z}} \cdot t = \omega(t=0) + \frac{T}{I_{z,z}} \cdot t \quad (7)$$

$$\varphi(t) = \varphi(t=0) + \frac{1}{2} \cdot \frac{m_\mathrm{a} g r_\mathrm{a}}{I_{z,z}} \cdot t^2 = \varphi(t=0) + \frac{1}{2}\frac{T}{I_{z,z}} \cdot t^2 \quad (8)$$

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Theory (7/7)

The potential energy of the accelerating weight, using eq. (8) and (7), is

$$E = m_\mathrm{a} g h(t) = -m_\mathrm{a} g \varphi(t) r_\mathrm{a} = \frac{1}{2} \cdot \frac{m_\mathrm{a}^2 g^2 r_\mathrm{a}^2}{I_{z,z}} \cdot t^2 = \frac{1}{2} I_{z,z} \omega^2$$

thus verifying (4).

If a weight m_i is mounted to a rod that can rotate around the fixed z-axis perpendicular to it in a distance r_i, maybe the rod lying along the y-axis, then the coordinates of the weight are $(0, r_i, 0)$ and, according to (3), the moment of inertia around the z-axis is $I_{z,z} = m_i r_i^2$, around the x-axis it is $I_{x,x} = m_i r_i^2$ and around the y-axis it is $I_{y,y} = 0$

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Equipment

Position	Material	Item No.	Quantity
1	Precision pivot bearing	02419-00	1
2	Tripod base PHYWE	02002-55	1
3	Inertia rod	02417-03	1
4	Turntable with angle scale	02417-02	1
5	Cobra SMARTsense - Rotary Motion (Bluetooth + USB)	12918-01	1
6	measureLAB, multi-user license	14580-61	1
7	Support rod, stainless steel, l = 250 mm, d = 10 mm	02031-00	1
8	Bench clamp expert	02011-00	1
9	Fish line, l. 100m	02090-00	1
10	Weight holder, 10 g	02204-00	1
11	Slotted weight, black, 10 g	02205-01	10
12	Slotted weight, black, 50 g	02206-01	2
13	USB charger for Cobra SMARTsense and Cobra4	07938-99	1

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Setup and Procedure

Setup

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Set-up the experiment as shown in Fig. 1: Connect the movement sensor to the computer. Adjust the turntable to be horizontal. Fix the silk thread (with the weight holder on one end) with the screw of the precision bearing or a piece of adhesive tape to the wheels with the grooves on the axis of rotation and wind it several times around one of the wheels – enough turns that the weight may reach the floor. Make sure that the thread, the wheel of the movement sensor and the selected wheel of the precision bearing are well aligned so that the thread runs horizontally between the wheels. Place the holding device with cable release below the turntable so that it holds the turntable when put in its hole, but does not disturb the movement after release.

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Procedure (1/4)

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Make sure that the thread, the wheel of the movement sensor and the selected wheel of the precision bearing are well aligned so that the thread runs horizontally between the wheels. Place the holding device with cable release below the turntable so that it holds the turntable when put in its hole, but does not disturb the movement after release.

Start the software and load the experiment. Start the measurement with the "record" button and release the turntable. Stop data recording just before the weight reaches the floor with the "stop" button (in order to not get irrelevant data at the end of your recorded data).

Fig. 2: Software measureLAB

Procedure (2/4)

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1. Record measurements with different accelerating weights m_a of up to 100 g. Let the thread run over the same wheel of the precision bearing (r = const) and the large wheel of the movement sensor ($r_{\mathrm{ms}} = 6\,\mathrm{mm}$) for the entire measurement series. Set the respective translation for the selected wheels from the "virtual channels" . When done, only one virtual channel must appear highlighted in green as shown in Fig. 2 (e.g. translation factor of 2.5 for the small wheel of the precision bearing (r = 15 mm)and the large wheel of the movement sensor ($r_{\mathrm{ms}} = 6\,\mathrm{mm}$) selected). Record measurements with the thread running over different wheels of the precision bearing. Use respective accelerating masses to maintain the torque $\vec{T} = \vec{r} \times \vec{F} = r \cdot m_{\mathrm{a}} \cdot g$ with $g = 9.81\,\mathrm{m/s^2} = 9.81\,\mathrm{N/kg}$being the gravitational acceleration. For example, use
   a) $m_{\mathrm{a}} = 60\,\mathrm{g}$ for r = 15 mm, b) $m_{\mathrm{a}} = 30\,\mathrm{g}$ for r = 30 mm, and
   c) $m_{\mathrm{a}} = 20\,\mathrm{g}$ for r = 45 mm.
   Each time, the torque $T = r \cdot m_{\mathrm{a}} \cdot g = 8.83\,\mathrm{mNm}$ will be constant.

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Procedure (3/4)

Record measurements with a constant weight m_a = const for each wheel of the precision bearing.

You may also take a measurement with the weight so light that it nearly does not affect the movement (like the empty weight holder of 10 g) but is enough to drive the turntable.

If necessary, start the turntable by a shove with your hand. Due to light friction, you should observe an almost unaccelerated movement.

Procedure (4/4)

2. Remove the turntable and mount the inertia rod to the precision bearing. Place its two weight holders symmetrically on the rod with both at the same distance to the axis. Take measurements with varying accelerating masses $m_{\mathrm{a},i}$ on one wheel groove with r = const and also with a constant accelerating mass m_a = const for all three wheel grooves with the radii r_i. Let the thread run over the large wheel of the movement sensor ($r_{\mathrm{ms}} = 6\,\mathrm{mm}$) and select the virtual channel with the respective translation for the used wheel of the precision bearing. Both masses on the bar must stay mounted symmetrically at the same positions. For further investigations, you can determine the behaviour with the masses at different symmetrical positions. For higher precision, you can also repeat the second part with a series of accelerating masses $m_{\mathrm{a},i}$ for each wheel groove.

Notes:
For heavy weights you may set the "Get value" time to 200 ms and for low weights you may set this time to higher values.
For further investigations, you can also use the small wheel of the movement sensor and repeat the measurement series. Make sure to select the respective translation.

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Evaluation

Results (1/4)

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Plot the angle of rotation vs. the square of time. The correct zero of time t_0 (the begin of movement without digital distortion) may be found with the "Regression" tool of measure. Select time as source channel and enter the operation $(t - t_0)^2$ with your actual value t_0 in your channel modification window. Then select the square of time as x-axis and "angle" as y-axis. The slope of the obtained curve is half the angular acceleration and may be determined with the "Regression" tool. Fig. 4 shows the obtained data of several measurements which were put into one diagram and scaled to the same value. The linearity vs. the square of time can be seen well.

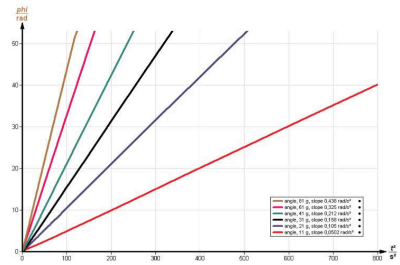

Fig. 4: Angle vs. square of time for the turntable.

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Results (2/4)

In Fig. 5, the angular acceleration values of Fig. 4 were plotted vs. the used torque $T = g m_\mathrm{a} r_\mathrm{a}$. The inverse slope is the moment of inertia and reads $I = 13.3\,\mathrm{mN\,m\,s^2} = 133\,\mathrm{kg\,cm^2}$ for the disc. You can also plot the moment of inertia $I_{z,z} = m_i r_i^2 + I_{\mathrm{rod};z,z}$ vs. the weight m_i.

With the regression values you will obtain r_i from the slope and $I_{\mathrm{rod};z,z}$ from the axis intercept. The angular acceleration data can be evaluated from the measurements with the "Regression" function. The moment of inertia values are calculated by virtual channels.

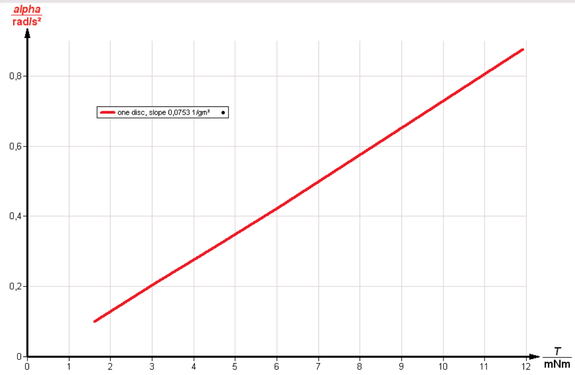

Fig. 5: Angular acceleration vs. accelerating torque.

Results (3/4)

Else you may evaluate the data using a bilogarithmic plot – but then it's crucial to correct the data for the right zero point of time and angle using virtual channels subtracting t_0 and adding/subtracting φ_0 and making a plot of the changed channels, erasing the values lower than zero in the data table and using the "Display options" tool to set the scaling of both axes to "logarithmic". Fig. 6 shows an example for the bar with 2 times 80 g mounted 21 cm away from the axis and accelerated with a weight of 21 g on the wheel groove with a radius of 15 mm, i.e. a torque of 3.09 mNm.

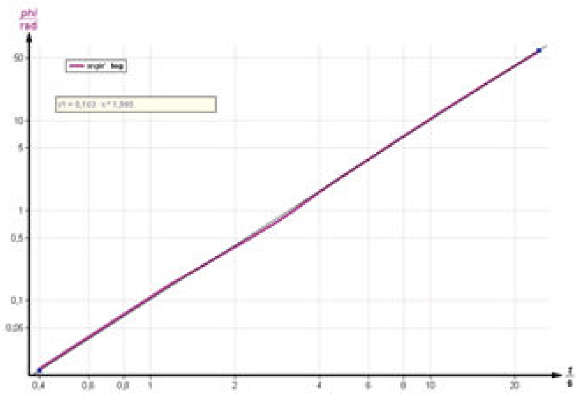

Fig. 6: Bilogarithmic plot of angle vs. time.

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Results (4/4)

Since $\varphi = \frac{1}{2}\alpha \cdot t^2$, the plot reads

$\frac{1}{2}\alpha = 0.103\,\frac{1}{\mathrm{s}^2} \Rightarrow \alpha = 0.203\,\frac{1}{\mathrm{s}^2}$

On the other hand T = 4.09 mNm: $\alpha \cdot \hat{I} \;\Rightarrow \hat{I}_z = 150\,\mathrm{kg\,cm}^2$ compared to theoretical

$\hat{I}_z = 2 \cdot 80\,\mathrm{g} \cdot (21\,\mathrm{cm})^2 + \hat{I}_{\mathrm{rod},z} = 70.6\,\mathrm{kg\,cm}^2 + 72\,\mathrm{kg\,cm}^2 = 143\,\mathrm{kg\,cm}^2$

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