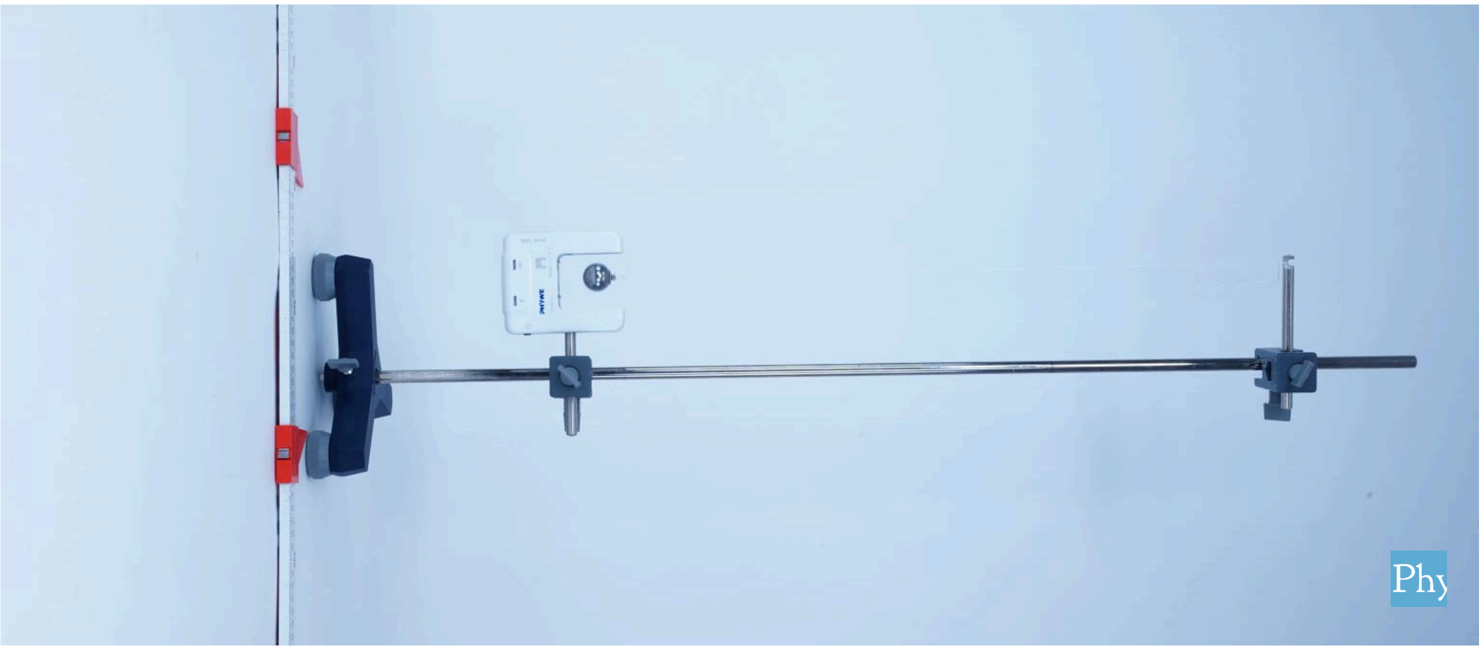
Mathematical pendulum with COBRA SMARTsense



Physics | Acoustics | Wave Motion

Difficulty level	Group size	Preparation time	Execution time
easy	2	10 minutes	10 minutes

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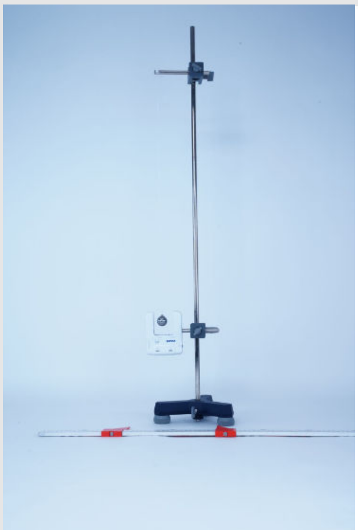
General information

Application



Setup

A pendulum is a body suspended from a fixed support so that it swings freely back and forth under the influence of gravity. When a pendulum is displaced sideways from its resting, equilibrium position, it is subject to a restoring force due to gravity that will accelerate it back toward the equilibrium position. When released, the restoring force acting on the pendulum's mass causes it to oscillate about the equilibrium position, swinging it back and forth.

The mathematics of pendulums are in general quite complicated. Simplifying assumptions can be made, which in the case of a simple pendulum allow the equations of motion to be solved analytically for small-angle oscillations.

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Other information (1/2)

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Prior knowledge

The prior knowledge required is found in the theory section.

Main principle

A mass, considered as of point form, suspended on a thread and subjected to the force of gravity, is deflected from its position of rest. The period of the oscillation thus produced is measured as a function of the thread length and the angle of deflection.

Other information (2/2)

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Learning objective

The goal of this experiment is to investigate the principle behind the mathematical pendulum.

Tasks

1. For small deflections, the oscillation period is determined as a function of the cord length.
2. The acceleration due to gravity is determined.
3. The oscillation period is determined as a function of the deflection.

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Theory (1/2)

From the energy equation there follows, with the notation of Fig. 1:

$\frac{1}{2}I\left(\frac{d\phi}{dt}\right)^2 + mgl(1 - cos\phi) = E_0 = \text{const.}$ (1)

Since the angular velocity vanishes at the reversal point, when $\phi = \alpha$

then one obtains for E_0

$E_0 = mgl(1 - \cos\alpha)$

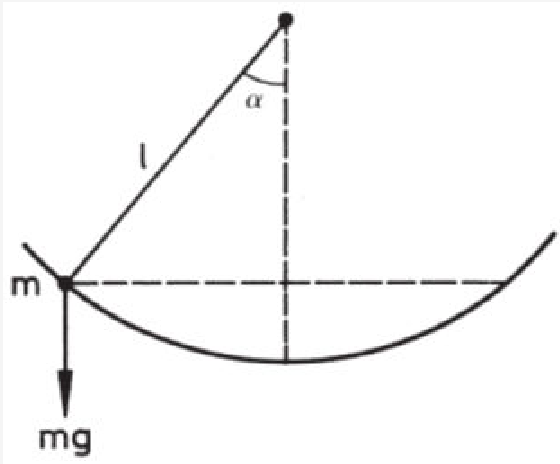

Fig. 1: Motion of the pendulum.

Theory (2/2)

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Therefore, from (1), with

$\int_0^{T/4} dt = T/4 = \sqrt{\frac{l}{g}\int_0^{\alpha}(\frac{d\phi}{\sqrt{2(\cos\phi - \cos\alpha)}})}$

with $I = ml^2$

With $k = \sin\alpha/2$, the period is obtained as

$T = 4\sqrt{\frac{l}{g}\int_0^{\pi/2}(\frac{d\phi}{\sqrt{1-k^2\sin^2\phi}})} = 4\sqrt{\frac{l}{g}}K(k)$

where K is the complete 1st-order elliptical integral.

Development of the series for gives

$T = 2\pi\sqrt{\frac{l}{g}}\left\{1 + \frac{1}{4}\sin^2\frac{\alpha}{2} + \ldots\right\}$ (2)

For small values of α ($\alpha < 2°$):

$T = 2\pi\sqrt{\frac{l}{g}} = \frac{2\pi}{\sqrt{g}} \cdot l^{1/2}$ (3)

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Equipment

Position	Material	Item No.	Quantity
1	Cobra SMARTsense - Photogate, 0 ... ∞ s (Bluetooth + USB)	12945-00	1
2	Steel ball with eyelet, d 25.4mm	02465-01	1
3	Steel ball with eyelet, d 32mm	02466-01	1
4	Meter scale, l = 1000 mm	03001-00	1
5	Cursors, 1 pair	02201-00	1
6	Fish line, l. 100m	02090-00	1
7	Boss head	02043-00	2
8	Clamping pads on stem	02050-00	1
9	Support rod, stainless steel 18/8, l = 1300 mm, d = 12 mm	02041-00	1
10	Tripod base PHYWE	02002-55	1

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Setup and Procedure

Setup and Procedure (1/2)

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The Cobra SMARTsense Photogate and measureAPP are required to perform the experiment. The app can be downloaded for free from the App Store - QR codes see below. Check whether Bluetooth is activated on your device (tablet, smartphone).

measureAPP for Android operating systems

measureAPP for iOS operating systems

measureAPP for Tablets / PCs with Windows 10

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Setup and Procedure (2/2)

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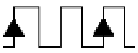
The experimental set up is arranged as shown in Fig. 2. The steel ball is tied to the fishing line and the latter is fixed in the clamping pads on stem. With a new line, the ball should be allowed to hang for a few minutes, since the fishing line stretches slightly. The pendulum length should be measured before and after the experiment and averaged in each case. The radius of the ball should be taken into account in the measurement. For problem 1, the light barrier can be used to measure a full cycle. To measure the oscillation period as a function of the deflection, switch the light barrier to the following position:



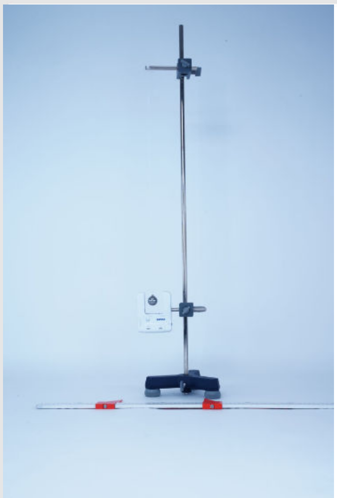


Fig. 2: Experimental setup

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Evaluation

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Results (1/2)

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From the regression line to the measured values of Fig. 3 with the exponential statement

$$Y = A \cdot X^B$$

the exponent is obtained

$$B = 0.502 \pm 0.001$$

and

$$A = 2.007 s/\sqrt{m} \Rightarrow g = (\frac{2\pi}{A})^2$$

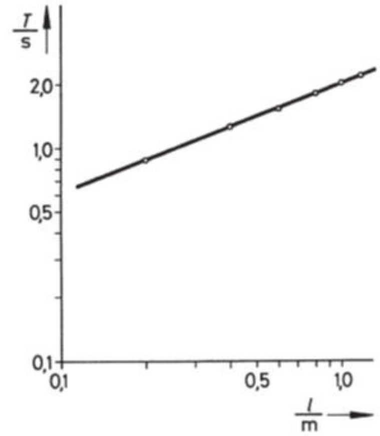

Fig. 3: Period of the pendulum as a function of length.

Results (2/2)

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From this, with (3), the value for the acceleration due to gravity is obtained as

$$g = 9.80 m/s^2$$

For larger angles α, T depends on α (2).

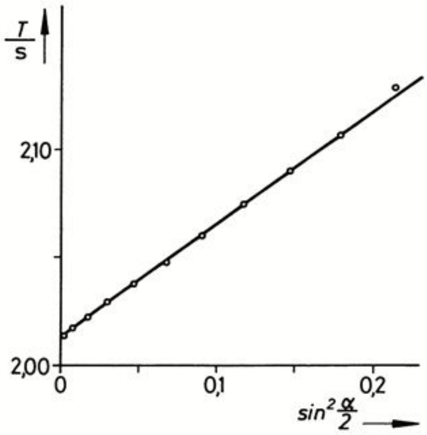

Fig. 4: Period of the pendulum as a function of the angle of deflection.

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