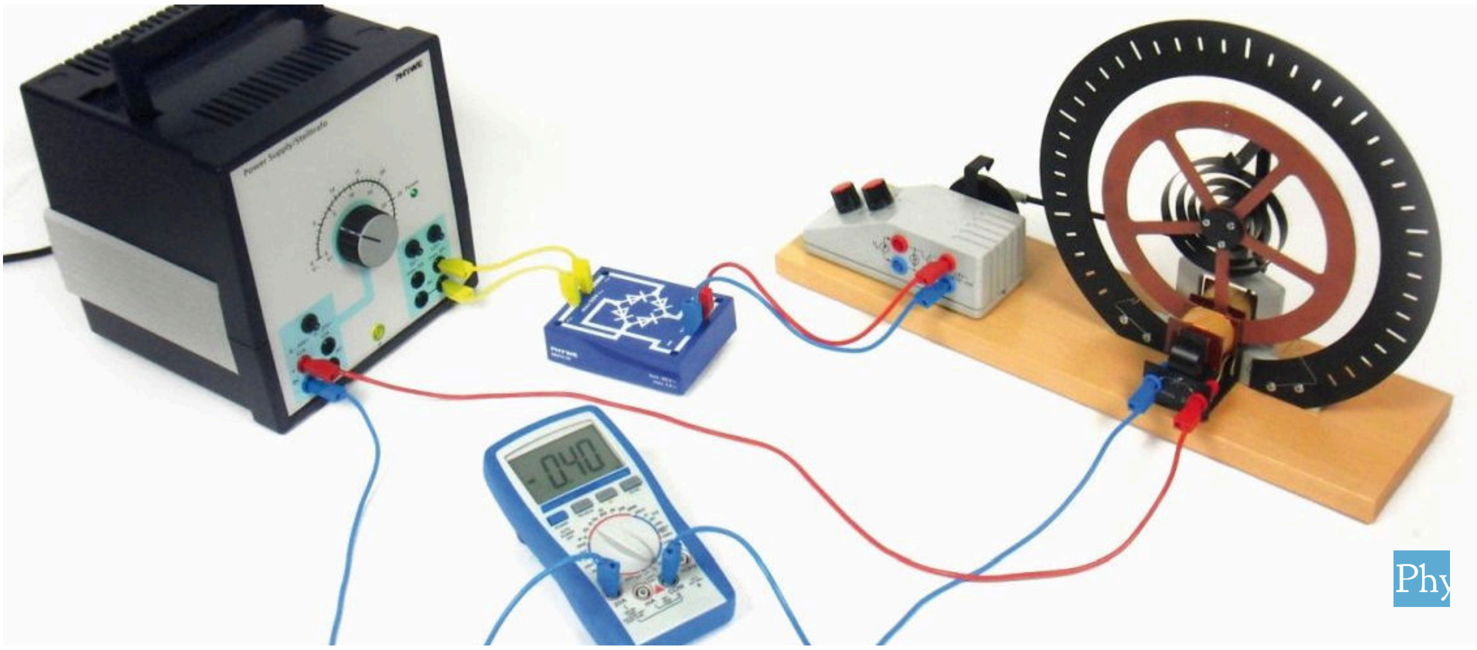
Forced oscillations - Pohl's pendulum with measure Dynamics



The goal of this experiment is to investigate the oscillation behaviour induced by forced oscillation.

Physics | Acoustics | Wave Motion

Difficulty level	Group size	Preparation time	Execution time
medium	-	-	-

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General information

Application

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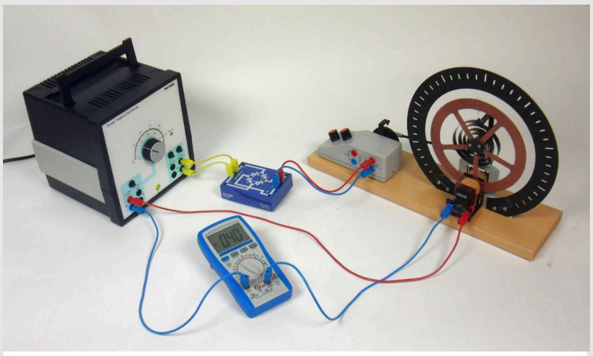

Experimental setup

Pendulum oscillations offer a first understanding of mechanical systems close to the harmonic oscillator, which is fundamental in the description of many physical systems in fields such as particle physics and solid state physics.

This experiment investigates the behaviour of a system, that is forced into oscillation. It offers insight in phenomena such as resonance.

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Robert-Bosch-Breite 10
37079 Göttingen

Tel.: 0551 604 - 0
Fax: 0551 604 - 107

info@phywe.de
www.phywe.de

Other information (1/3)

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Prior knowledge

The behaviour of a singular pendulum should be known.

Main principle

If an oscillating system is allowed to oscillate freely, we can observe that the decrement of successive maximum amplitudes strongly depends on the damping value. If the oscillating system is caused to oscillate by an external torsional oscillation, we can observe that the amplitude in a stationary state is a function of the frequency and amplitude of the external periodic torsional oscillation and of the damping value. The aim of this experiment is to determine the characteristic frequency of the free oscillation as well as the resonance curve of a forced oscillation.

Other information (2/3)

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Learning objective

The goal of this experiment is to investigate the oscillation behaviour induced by forced oscillation.

Tasks

Free oscillation

1. Determine the oscillating period and the characteristic frequency of the undamped case.
2. Determine the oscillating periods and the corresponding characteristic frequencies for different damping values. The corresponding ratios of attenuation, the damping constants and the logarithmic decrements are to be calculated.
3. Realize the aperiodic and the creeping case.

PHYWE excellence in science
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Other information (3/3)

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Tasks

Forced oscillation

1. Determine the resonance curves and represent them graphically using the damping values of A. Determine the corresponding resonance frequencies and compare them with the resonance frequency values found beforehand.
2. Observe the phase shifting between the torsion pendulum and the stimulating external torque for a small damping value for different stimulating frequencies

Theory (1/3)

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Undamped and damped free oscillation

In case of free and damped torsional vibration torques M_1 (spiral spring) and M_2 (eddy current brake) act on the pendulum. We have

$M_1 = -D^0\Phi$ and $M_2 = -C\dot{\Phi}$

Φ = angle of rotation. $\dot{\Phi}$ = angular velocity, D^0 = torque per unit angle, C = factor of proportionality depending on the current which supplies the eddy current brake.

The resultant torque $M_1 = -D^0\Phi - C\dot{\Phi}$

leads us to the following equation of motion:

$I\ddot{\Phi} + c\dot{\Phi} + D^0\Phi = 0$ (1)

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Robert-Bosch-Breite 10
37079 Göttingen

Tel.: 0551 604 - 0
Fax: 0551 604 - 107

info@phywe.de
www.phywe.de

Theory (2/3)

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Dividing Eq. (1) by and using the abbreviations

$\delta = \frac{C}{2I}$ and $\omega_0^2 = \frac{D^0}{I}$

results in $\ddot{\Phi} + 2\delta\Phi + \omega_0^2\Phi = 0$ (2)

δ is called the “damping constant” and

$\omega_0 = \sqrt{\frac{D^0}{I}}$

$F = m\omega^2 r$

the characteristic frequency of the undamped system.

The solution of the differential equation (2) is

$\Phi(t) = \Phi_0 e^{-\delta t} \cos \omega t$ (3)

$\omega = \sqrt{\omega_0^2 - \delta^2}$ (4)

Theory (3/3)

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Forced oscillation

If the pendulum is acted on by a periodic torque $M_a = M_0 \cos \omega_a t$ Eq. (2) changes into

$\ddot{\Phi} + 2\delta\dot{\Phi} + \omega_0^2\Phi = F_0 \cos \omega_a t$ (7)

where $F_0 = \frac{M_0}{I}$

In the steady state, the solution of this differential equation is

$\Phi(t) = \Phi_a \cos(\omega_a t - \alpha$ (8)

where

$\Phi_\alpha = \frac{\Phi_0}{\sqrt{\left(1-\left(\frac{\omega_\alpha}{\omega_0}\right)^2\right)^2+\left(2\frac{\delta}{\omega_0}\frac{\omega_\alpha}{\omega_0}\right)^2}}$ (9)

and $\Phi_\alpha = \frac{F_0}{\omega_0^2}$

Furthermore:

$\tan \alpha = \frac{2\delta\omega_\alpha}{\omega_0^2 - \omega_a^2}$ (10)

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Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Equipment

Position	Material	Item No.	Quantity
1	Torsion pendulum after Pohl	11214-00	1
2	Digital Laboratory Power Supply 2 x 0 - 30 V/0 - 5 A DC/5 V/3 A fixed	EAK-P-6145	1
3	Connecting cord, 32 A, 750 mm, red	07362-01	2
4	Connecting cord, 32 A, 750 mm, blue	07362-04	2
5	Software "Measure Dynamics", single user	14440-61	1

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Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
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info@phywe.de
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Setup and Procedure

Setup (1/3)

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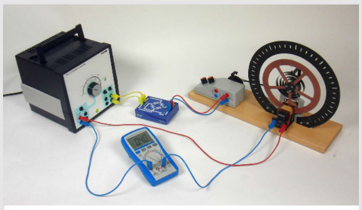

Fig. 1: Experimental setup

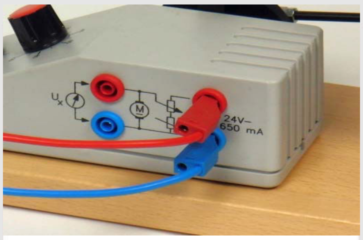

Fig. 3: DC motor

The experiment is set up as shown in Fig. 1 und 2. The DC output of the power supply unit is connected to the eddy current brake. The motor also needs DC voltage. For this reason, a rectifier is inserted between the AC output (12 V) of the power supply unit and to the two right sockets of the DC motor (see Fig. 3). The DC current supplied to the eddy current brake, I_B, is set with the adjusting knob of the power supply and is indicated by the ammeter.

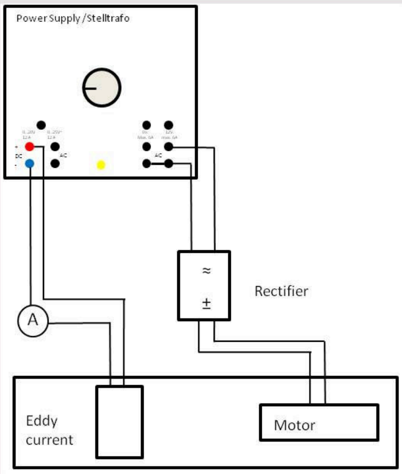

Fig. 2: Electrical connection of the experiment.

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Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Setup (2/3)

In terms of the video that will be recorded, the following settings and positioning of the camera must be considered:

- Set the number of frames per second to approximately 30 fps.
- Select a light-coloured, homogeneous background.
- Provide additional lighting for the experiment.
- The experiment set-up, i.e. Pohl's pendulum, should be in the centre of the video. To ensure this, position the video camera on a tripod centrally in front of the experiment set-up. For the experiment concerning the forced oscillations, however, it is important that the excitation motor is also filmed. However, the pendulum must remain in the centre of the video.

Setup (3/3)

- The experiment set-up should fill the video image as completely as possible.
- The optical axis of the camera must be parallel to the experiment set-up.
- For the video analysis, we recommend gluing a circular, coloured sheet of paper onto the rotating needle of the pendulum (see Figure 4).
- Measure the width of the pendulum for scaling.

Then, the video recording process and the experiment can be started.

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Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Procedure (1/5)

A. Free oscillation

1. Determine the oscillating period and the characteristic frequency of the undamped case

To determine the characteristic frequency ω_0 of the torsion pendulum without damping (I_B = 0),

- start the video recording process,
- deflect the pendulum completely to one side
- measure the time for several oscillations.
- stop the video recording process,

The measurement is to be repeated several times and the mean value of the characteristic period $\bar{T}_0$ is to be calculated.

Procedure (2/5)

2. Determine the oscillating periods and the corresponding characteristic frequencies for different damping values.

Prior to starting the experiment, it must be ensured that the pointer of the pendulum at rest coincides with the zero position of the scale (see Figure 4). This can be achieved by manually turning the eccentric disc of the motor.

In the same way the characteristic frequencies for the damped oscillations are found using the following current intensities for the eddy current brake:

$I_B \sim 0.45\,\mathrm{A}$

$I_B \sim 0.80\,\mathrm{A}$

The damping values can then be determined based on the recorded video.

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Robert-Bosch-Breite 10
37079 Göttingen
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Fax: 0551 604 - 107
info@phywe.de
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Procedure (3/5)

3. Realize the aperiodic and the creeping case

To realize the aperiodic case ($I_B \sim$ 2.0 A) and the creeping case ($I_B \sim$ 2.3 A) the eddy current brake is briefly loaded with more than 2.0 A. **Caution**: Do not use current intensities above 2.0 A for the eddy current brake for more than a few minutes.

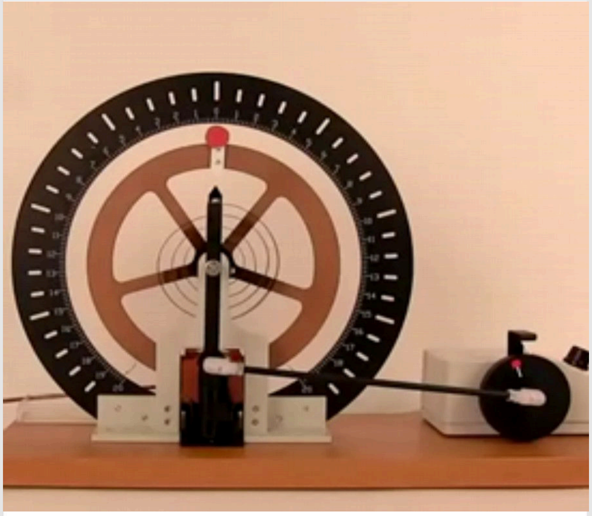

Fig. 4: Pendulum pointer at zero-position.

Procedure (4/5)

B. Forced oscillation

To stimulate the torsion pendulum, the connecting rod of the motor is fixed to the upper third of the stimulating source. The stimulating frequency ω_α of the motor can be found by using a stopwatch and counting the number of turns (for example: stop the time of 10 turns).

1. Determine the resonance curves and represent them graphically using the damping values of A.

The measurement begins with small stimulating frequencies ω_α. ω_α is increased by means of the motorpotentiometer setting “coarse“. In the vicinity of the maximum ω_α is changed in small steps using the potentiometer setting “fine“ (see Fig. 5). In each case, readings should only be taken after a stable pendulum amplitude has been established. In the absence of damping or for only very small damping values, ω_α must be chosen in such a way that the pendulum does not exceed its scale range.

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Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
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Procedure (5/5)

Fig. 5: Control knobs to set the motor potentiometer. Upper knob: “coarse”; lower knob: “fine”.

2. Observe the phase shifting between the torsion pendulum and the stimulating external torque for a small damping value for different stimulating frequencies

Chose a small damping value and stimulate the pendulum in one case with a frequency ω_α far below the resonance frequency and in the other case far above it. Observe the corresponding phase shifts between the torsion pendulum and the external torque. In each case, readings should only be taken after a stable pendulum amplitude has been established.

Evaluation

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Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
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Evaluation (1/13)

Transfer the various videos that have been recorded to the computer. Then, start "measure Dynamics" and open the video under "File" – "Open video ...". Mark the start of the experiment ("Start selection" and "Time zero") and the end of the experiment ("End selection") in the video for further analysis via the menu line above the video.

The experiment begins when the torsional pendulum is released and it ends when the video ends. Mark the width of Pohl's pendulum with the scale that appears in the video by way of "Video analysis" – "Scaling ..." – "Calibration" and enter the measured length (in this case 0.30 m) into the input window. In addition, enter the frame rate that has been set for the recording process under "Change frame rate" and position the origin of the system of coordinates in the centre of rotation of the pendulum under "Origin and direction".

Evaluation (2/13)

Then, the actual motion analysis can be started under "Video analysis" – "Automatic analysis" or "Manual analysis". For the automatic analysis, we recommend selecting "Motion and colour analysis" on the "Analysis" tab. Under "Options", the automatic analysis can be optimised, if necessary, e.g. by changing the sensitivity or by limiting the detection radius. Then, look for a film position in the video where the object that is to be analysed is perfectly visible. Click the object. If the system recognises the object, a green rectangle appears and the analysis can be started by clicking "Start".

If the automatic analysis does not lead to any satisfying results, the series of measurements can be corrected under "Manual analysis" by manually marking the object that is to be analysed.

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Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Evaluation (3/13)

In order to determine the average period of oscillation $\bar{T}_0$ and the corresponding characteristic frequency $\bar{\omega}_0$ of the free and undamped, oscillating torsional pendulum, we have to determine the time that the pendulum needs for several oscillations (e.g. 10 oscillations). This leads to:

$\bar{T}_0 = 1.962\,\mathrm{s}$

$\bar{\omega}_0 = 3.20\,\mathrm{s}^{-1}$

Evaluation (4/13)

The average period of oscillation $\bar{T}_0$ and the average frequency $\bar{\omega}_0$ are determined in the same manner as for task 1. For $I_{B,1} = 0.45\,\mathrm{A}$, this leads to

$\bar{T}_0 = 1.946\,\mathrm{s}$

$\bar{\omega}_0 = 3.23\,\mathrm{s}^{-1}$

For $I_{B,1} = 0.80\,\mathrm{A}$, this leads to

$\bar{T}_0 = 1.935\,\mathrm{s}$

$\bar{\omega}_0 = 3.25\,\mathrm{s}^{-1}$

Then, perform a graphical representation of the maximum amplitudes on one side of the pendulum, i.e. always on the right-hand side or always on the left-hand side of the pendulum, as a function of time. Add a new column to the worksheet by clicking "New column" in the table menu line. Enter "Phi" (unit: "°"; formula: "arctan2(x;y)*360/(2*π)") into this new column. Then, create another column (name: "Phi_max"; unit: "°"). Find the various maxima of "Phi" in the table and enter the values of "Phi" of these maxima into the column "Phi_max". In order to display the maxima of "Phi" as a function of time, select "Display" and "Diagram", click "Options", delete all of the already existing graphs, and select the graphs t (horizontal axis) – "Phi_max" (vertical axis). For $I_{B,1} = 0.45\,\mathrm{A}$, this leads to:

PHYWE excellence in science
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
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Evaluation (5/13)

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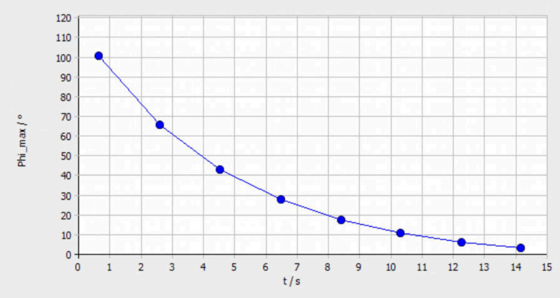

For $I_{B,1} = 0.80\,\mathrm{A}$, this leads to:

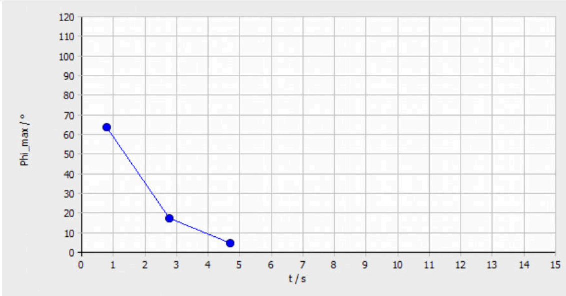

Fig. 6: Representation of the maximum deflections of the pendulum for $I_{B,1}$ = 0.45 A on one side as a function of time t.

Fig. 7: Representation of the maximum deflections of the pendulum for $I_{B,1}$ = 0.80 A on one side as a function of time t.

Evaluation (6/13)

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We already know from the theory – and Figures 6 and 7 confirm this – that a higher current I_B causes the torsional pendulum to be decelerated more quickly. The resulting relationship between damping, damping constant K, and logarithmic decrement Λ is calculated as follows:

Eq. (3) shows that the amplitude $\varphi(t)$ of the damped oscillation has decreased to the e^{th} part of the initial amplitude φ_0 after the time $t = 1/\delta$ has elapsed. Moreover, from Eq. (3) it follows that the ratio of two successive amplitudes is constant.

$$\frac{\varphi_n}{\varphi_{n+1}} = K = e^{\delta T} \quad (5)$$

K is called the “damping ratio”, T the oscillating period, and the quantity

$$\Lambda = \ln K = \delta T = \ln \frac{\varphi_n}{\varphi_{n+1}} \quad (6)$$

is called the “logarithmic decrement“.

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Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
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Evaluation (7/13)

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Sample results for the characteristic damping values:

I [A]	$1/\delta$ [s]	δ [1/s]	$\omega = \sqrt{\omega_0^2 - \delta^2}$ [1/s]	$K = \frac{\Phi_n}{\Phi_{n+1}}$	Λ
0.45	4.6	0.33	3.21	1.54	0.43
0.80	1.5	0.67	3.27	3.72	1.31

The difference of the values for the frequency ω is due to the fact that δ can be determined only in a rather imprecise manner. Still, the calculated frequencies are nearly consistent with the measured frequencies.

Evaluation (8/13)

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Add a new column with the name "Phi" to the table in the same manner as for task A2.

Select "Display" and "Diagram", click "Options", delete all of the already existing graphs, and select the graphs t (horizontal axis) – "Phi" (vertical axis). This leads to:

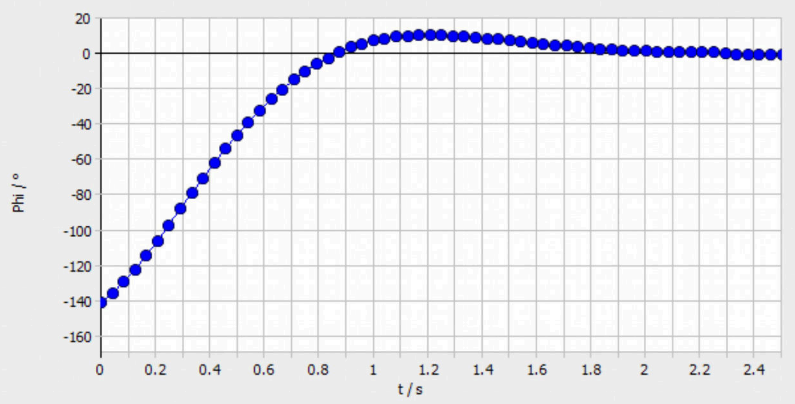

Fig. 8: Representation of the deflections of the pendulum for $I_{B,1}$ = 2.00 A as a function of time t.

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Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Evaluation (9/13)

Eq. (4) has a real solution only if $\omega_0^2 \geq \delta^2$. For $\omega_0^2 = \delta^2$, the pendulum returns in a minimum of time to its initial position without oscillating (aperiodic case).

For $\omega_0^2 \leq \delta^2$, the pendulum returns asymptotically to its initial position (creeping).

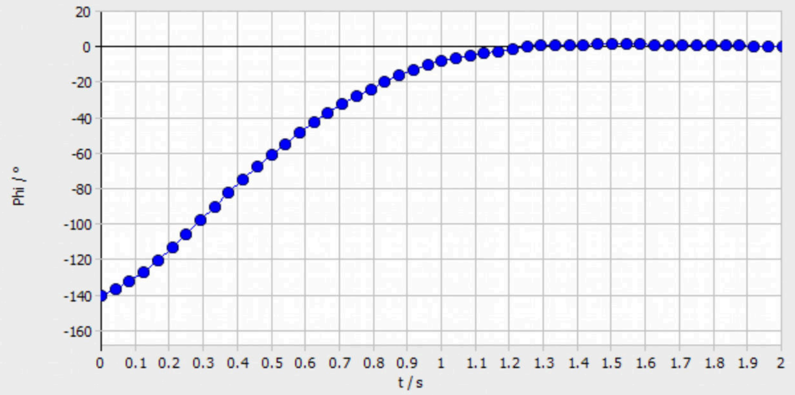

Fig. 9: Representation of the deflections of the pendulum for $I_{B,1}$ = 2.30 A as a function of time t.

Evaluation (10/13)

For this task, we recommend performing the analysis of the rotational motion manually, since only a few moments of the video area actually relevant for the evaluation. Add new columns for "Phi" (see task A2), for the period of oscillation "T" (unit: "s"), and for the excitation frequency "wa" (unit: "1/s", formula: "2*π/T")) to the worksheet. It may be necessary to add a minus sign to the formula when creating the "Phi" column so that the values become positive. Then, determine the excitation frequency (in the same manner as for task A1, but in this case observe the eccentric disc and not the pendulum). Wait until the torsional pendulum has developed a constant amplitude and mark the pendulum at the maximum in the manual analysis. Enter the period of oscillation, which has already been determined, into the same line under "T". Do the same for the other excitation frequencies.

In order to display the maximum deflection "Phi" as a function of the excitation frequency, select "Display" and "Diagram", click "Options", delete all of the already existing graphs, and select the graphs (horizontal axis) – "Phi" (vertical axis). This leads to:

PHYWE excellence in science
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
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Evaluation (11/13)

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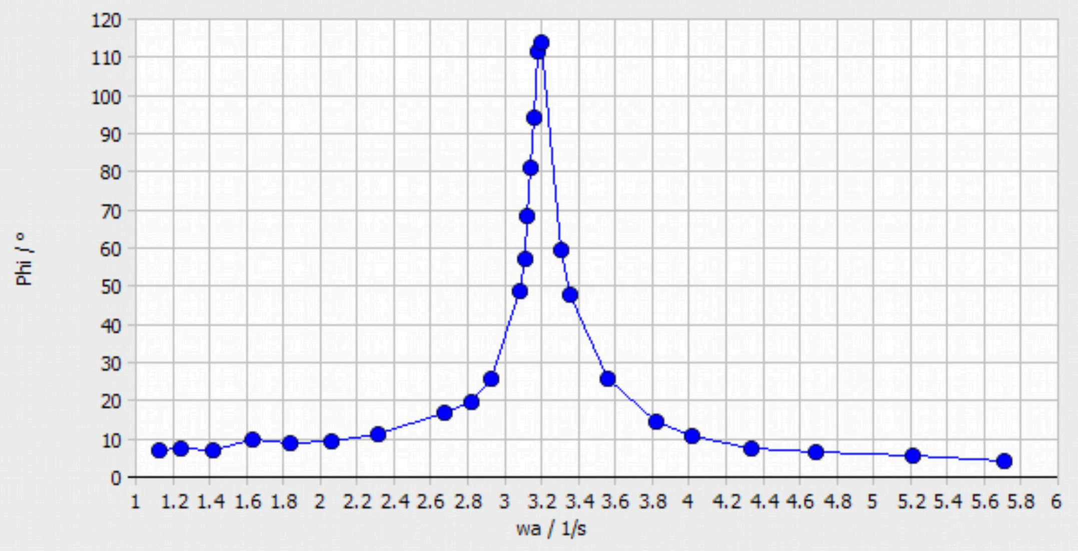
Fig. 10: Resonance curve for I_B = 0.23 A.



Evaluation (12/13)

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Figure 10 shows that the amplitude is very small in the case of low excitation frequencies. When the excitation frequency is slowly increased, a maximum can be observed. Increasing the excitation frequency further leads to smaller amplitudes until the amplitude drops nearly to 0. This is the characteristic shape of a resonance curve.

The relatively sharp maximum of the amplitude occurs at the so-called resonance frequency. Here, the resonance frequency is $\omega = 3.20\,\mathrm{s}^{-1}$. In addition, it becomes clear that the resonance frequency matches the characteristic frequency of the torsional pendulum, which has been determined in task A1. This gives rise to the suspicion that $\varphi \rightarrow \varphi_{\max}$ for $\omega_\alpha \approx \omega_0$. If one considers equation (9), this suspicion is confirmed by the theory. For higher damping values, the curve is above the represented curve, and it is below the represented curve for smaller damping values.

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Robert-Bosch-Breite 10
37079 Göttingen

Tel.: 0551 604 - 0
Fax: 0551 604 - 107

info@phywe.de
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Evaluation (13/13)

For this task, the video of task B1 is required. If we observe the torsional pendulum for a lower excitation frequency ω_a, it becomes clear that there is no phase difference between the torsional pendulum and exciting pendulum, i.e. the pendulum and the exciting torque are "in phase".

If the excitation frequency ω_a is much higher than the resonance frequency ω_0, the pendulum and the exciting torque show a phase difference π, i.e. they are in "antiphase" with each other. If the resonance frequency is applied to the torsional pendulum, the phase difference is $\pi/2$. The smaller the damping value is, the quicker is the transition from "in phase" oscillation to "antiphase" oscillation.

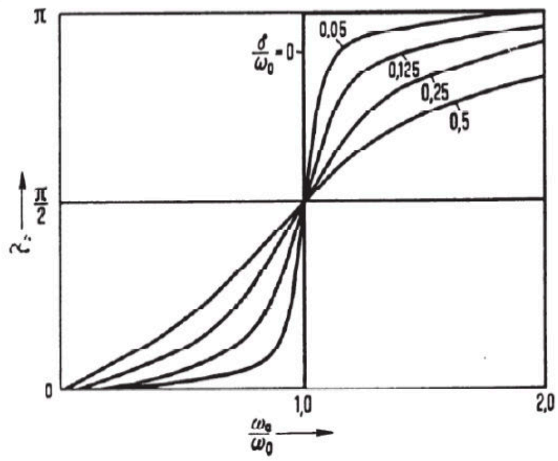

Fig 11: Phase difference in the case of a forced oscillation for different damping values.

Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de