Phase velocity of standing waves with Cobra SMARTsense







General information

Application



A standing wave is a wave which oscillates in time but whose peak amplitude profile does not move in space. The location at which the absolute value of the amplitude is minimum are called 'nodes', whereas the locations where the absolute value of the amplitude is maximum are called 'antinodes'. This phenomenon results from the interference between two waves traveling in opposite directions. The distance from one node to its neighbouring node equals half the wavelength $\lambda/2$.

The interference of waves is an absolute fundamental principle in physics.



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Other information (1/2)



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Prior knowledge



Basic knowledge of physical quantities such as frequency, wavelength and force as well as the theoretical description of a harmonic oscillation and waves in general should be available. Ideally, terms like phase velocity and group velocity as well as constructive and destructive interference should already be worked out theoretically.

Scientific principle



A linearly polarized standing transverse wave is generated on a rubber band with a square cross-section by means of a vibration generator. The wavelength is dependend on both, the excitation frequency and the phase velocity of the rope wave. The latter is varied by changing the tensile stress.

Other information (2/2)





Tasks



After the successful completion of this experiment you will be able to determine the phase velocity of a vibrating rubber band experimentally from the wavelength of a standing wave at a give excitation frequency. You will understand in which way the tensile stress of the rope affects the phase velocity.

- 1. At a constant tensile stress the wavelength λ of the wave propagating on the rope is determined with respect to the excitation frequency f. The phase velocity c is determined via the relation between frequency and wavelength.
- 2. The dependence of the phase velocity c of the rope wave from the tensile stress of the rope, is to be measured. The phase velocity is displayed as a function of the tensile stress.

Safety instructions





The general instructions for safe experimentation in science lessons apply to this experiment.

Theory (1/4)



A wave is an oscillation, which propagates in space and time and usually periodically through matter and space. One differentiates between transverse and longitudinal waves. In the case of transverse waves, the oscillation is perpendicular to the direction of the propagation of the wave. A simple example of a wave is a harmonic wave, as represented below.



A wave is characterized by its amplitude, the phase and its frequency f. In case of a harmonic wave, the wave length is called λ and represents the distance between two neighbouring maxima. For traversing this distance, it requires the same time as for a full oscillation T in a single section. The velocity (phase velocity $c(\lambda)$) determined on this basis amounts to

$$c = rac{\lambda}{T} = \lambda \cdot f$$



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Theory (2/4)

In this experiment, a periodic wave is produced, which propogates along the rubber rope and is reflected back at the support rod. The wave travelling in both directions, superimpose on each other, to form a standing wave with nodes (areas with rarely any vibrational amplitudes) and antinodes (area of the large vibrational amplitudes). The distance between wave nodes equals half the wave length ($\lambda/2$).

The phase velocity depends on the applied tensile stress σ of the rubber rope and is therefore linearly dependend on the applied force F and reciprocal proportional to the cross sectional area A:

 $\sigma = F/A$

The nominal cross sectional area of the rubber rope is $1.5 mm \times 1.5 mm$ while the mass density of rubber lies between about $0.9...1.0 g/cm^3$. The decrease in the cross sectional area A of the rubber rope, on account of the elongation of the rope can be neglected.

Theory (3/4)

The applied forces at the section can act tangentially on the rope. As the rope is deflected an effective perpendicular component $\overrightarrow{F_D}$ is produced for that section from the sum of the applied tension forces $\overrightarrow{F_A}$ and $\overrightarrow{F_R}$. Both forces are opposing with an included small angular difference $d\Theta$.



The resulting accelerating force is $|\overrightarrow{F_D}| = |\overrightarrow{F_A} - \overrightarrow{F_R}| = 2|\overrightarrow{F_A}| \cdot \sin(d\Theta/2)$ $\approx |\overrightarrow{F_A}| \cdot d\Theta \approx |\overrightarrow{F_A}| \cdot y \, dx$

which leads to an acceleration in y-direction of the section of the rope of mass m.





Theory (4/4)

Thereby $|\overrightarrow{F_A}| \equiv F$ is the average force applied to the rope along the rope and the net force resulting in an accelerating force F_y in *y*-direction can be expressed as:

$$|\stackrel{
ightarrow}{F_D}|pprox F_y=m\cdot\ddot{y}=F\cdot y\,dx$$

On the other hand the mass of the respective section is dependend on its cross sectional area A, its infinitesimal length dx and its density of mass ρ :

$$m = A \cdot dx \cdot \rho$$

Also including the rope tension $\sigma = F/A$, this leads to the differential equation

$$\ddot{y} = \frac{\sigma}{
ho} \cdot y$$

Which can be solved by the general solution

$$y=f(x\pm ct)$$
 with $c=\sqrt{rac{\sigma}{
ho}}$

where *c* is the phase velocity of propagation of the rope wave.

Equipment

Position	Material	Item No.	Quantity
1	PHYWE Digital Function Generator, USB	13654-99	1
2	Cobra SMARTsense - Force and Acceleration, \pm 50N / \pm 16g (Bluetooth + USB)	12943-00	1
3	External vibration generator for PHYWE Ripple Tank	11260-10	1
4	Connecting cord, 32 A, 1000 mm, yellow	07363-02	2
5	Tripod base PHYWE	02002-55	1
6	Support rod, stainless steel, 750 mm	02033-00	1
7	Square section rubber strip,I 10m	03989-00	1
8	Measuring tape, I = 2 m	09936-00	1
9	measureAPP - the free measurement software for all devices and operating systems	14581-61	1

Optional additional equipment

Position Material

Quantity

1

1 Digital Stroboscope (e.g. 21809-93)





Setup and procedure



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Setup (1/3)

Cobra SMARTsense force & acceleration and the measureAPP are required to measure the tensile stress. The app can be downloaded for free from the App Store - QR codes see below. Check that Bluetooth is activated on your device (tablet, smartphone).



measureAPP for Android operating systems



measureAPP for iOS operating systems



measureAPP for Tablets / PCs with Windows 10

Setup (2/3)



- Connect the digital function generator to the vibration generator. You can turn on the function generator, but make sure that the output amplitude is inicially set to zero.
- Fix the support rod in the tripod and mount the Cobra SMARTsense force & acceleration onto it at the same height as the lever of the vibration generator.
- Take a piece of a little bit more than 0,5 m of the rubber rope. Knot one of each ends to the sensor's hook and the lever of the vibration generator.





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Setup (3/3)



- Make sure to have force applied to the sensor. Turn on the sensor by pressing the I/O button for at least three seconds (the red blinking LED will confirm, that the sensor is switched on).
- Start the measureAPP. Select the Cobra SMARTsense force & acceleration sensor, which will appear in the list of sensors.
- Select the sensor mode 'Force'.
- Once the mode is chosen, your measureAPP should look like shown to the left.
- Use the 'set to zero' option.

Procedure (1/4)



- Turn on the digital function generator. Use the buttons in the middle to select the different properties. Increase and decrease the values by rotating the knob on the upper right side. The buttons below the knob (arrows left and right) can be used to select different digits of the respective value and thus to increase in decrease the increment.
- \circ Set the signal to sine curve and set the frequency to 50 Hz.
- $\circ~$ Increase the distance between the two ends of the rubber rope until you reach about $0.5\,N.$
- $\circ~$ Then increase the output voltage to about $6\,V.$

Procedure (2/4)

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Before starting the actual experiment, you should perform a semi-qualitative experiment as discribed below in order to get a feeling, in which manner the frequency of the rope wave depends on the reciprocal wave length $(1/\lambda)$.

- Modify the distance of the sensor relative to the vibrating lever by moving the tripod base, until you clearly see five antinodes. Accordingly you should find four sharp nodes.
- $\circ~$ Then vary the excitation frequency in steps of 10~Hz between 10~Hz and 80~Hz and observe the motion of the rubber band.



Procedure (3/4)



- Set the output of the function generator to zero.
- Place the sensor now at a position, such that the resulting force due to tensile stress is at 0.5 N. You can roughly determine the acting force with the help of the digital display. In order to precisely measure the force start a measurement and stop it after several seconds. Use the 'Average' tool to determine the mean value of the force. Adjust the position of the tripos slightly if necessary.



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Procedure (4/4)

Once the force for the tensile stress is set, increase the output amplitude of the function generator back to 6 V. Vary the frequency f between 10 Hz and 80 Hz until you find a certain amount of antinodes with sharp nodes in between. Measure the distance $\lambda/2$ between two neighboring nodes and note it together with the value for the frequency in the evaluation section.



- Find at least two more pairs of frequency and half wavelength and note them down as well.
- Set the output of the function generator back to zero.
- Repeat the complete experiment for at least four different applied forces (i.e. 0.75 *N*, 1.00 *N*, 1.25 *N*, 1.50 *N*).

Evaluation (1/3) Table 1

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Note the measured frequencies f and their corresponding half wavelength values λ/s for each applied force F in the tables below. Then calculate the respective phase velocities c and their mean value $\langle c \rangle$.





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Evaluation (3/3)

Calculate the square of the resulting mean phase velocities and plot it in dependence of the applied force. The slope of a linear regression $c^2 = m \cdot F$ then equals the reciprocal product of the mass density ρ and cross sectional area A of the rubber rope: $m = 1/(\rho A)$.





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