RLC measuring bridge (Item No.: P2441005)

Curricular Relevance

Area of Expertise: Physics
Education Level: University
Topic: Electricity and Magnetism
Subtopic: Inductance, Electromagnetic Oscillations, AC Circuits
Experiment: RLC measuring bridge

Difficulty	Preparation Time	Execution Time	Recommended Group Size
Difficult	1 Hour	2 Hours	2 Students

Additional Requirements:

Experiment Variations:

Keywords:

Wheatstone bridge, inductive and capacitive reactance, ohmic resistance, impedance, Kirchhoff's laws

Introduction

Overview

Ohmic resistances, inductances and capacitances are determined in a Wheatstone bridge circuit operated on AC. Balancing is done aurally through headphones, using the high sensitivity of the human ear.

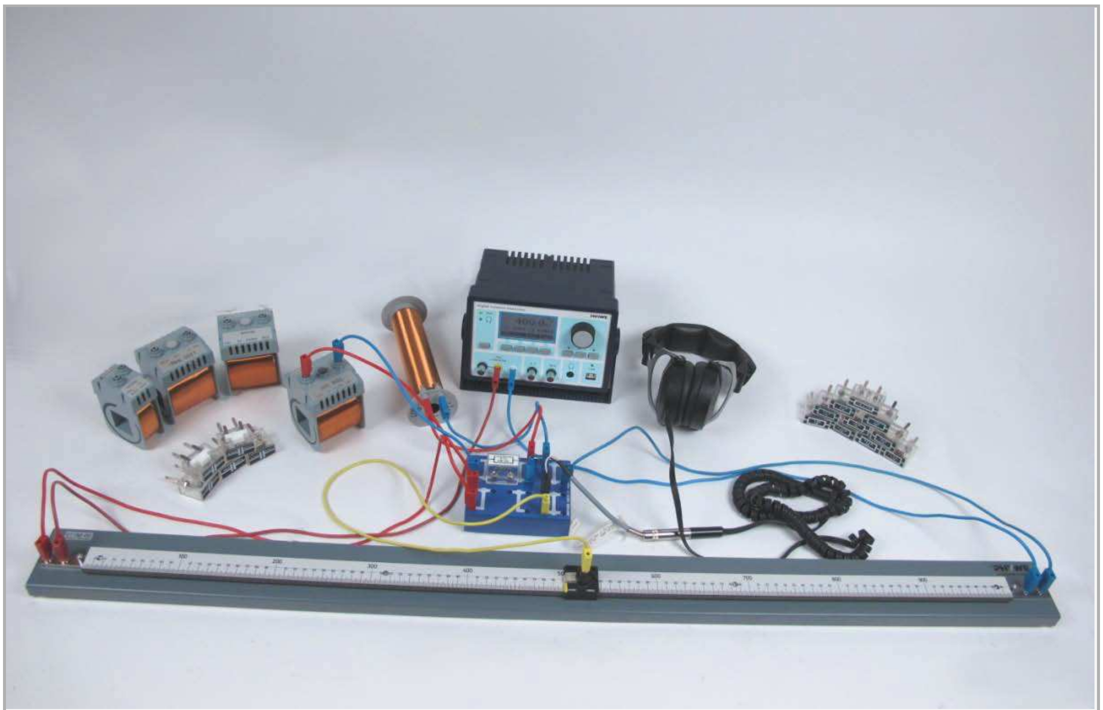

Fig. 1: Experimental set-up for determining unknown inductances

Equipment

Position No.	Material	Order No.	Quantity
1	PHYWE Digital Function Generator, USB	13654-99	1
2	Slide wire measurement bridge	07182-00	1
3	Headphone, stereo	65974-00	1
4	Coil, 300 turns	06513-01	1
5	Coil, 600 turns	06514-01	1
6	Coil, 1200 turns	06515-01	1
7	Coil, 600 turns, short	06522-01	1
8	Induction coil,300 turns,dia.40mm	11006-01	1
9	Resistor 1 Ohm 2%, 2W, G1	06055-10	1
10	Resistor 2 Ohm 2%, 2W, G1	06055-20	1
11	Resistor 5 Ohm 5%, 2W, G1	06055-50	1
12	Resistor 10 Ohm 2%, 2W, G1	06056-10	1
13	Resistor 330 Ohm, 1W, G1	39104-13	1
14	Resistor 470 Ohm, 1W, G1	39104-15	1
15	Resistor 680 Ohm, 1W, G1	39104-17	1
16	Resistor 1 kOhm, 1W, G1	39104-19	2
17	Resistor 1,5 kOhm, 1W, G1	39104-21	1
18	Resistor 2.2 kOhm, 1W, G1	39104-23	1
19	Resistor 3.3 kOhm, 1W, G1	39104-25	1
20	Capacitor 100pF/100V, G1	39105-04	1
21	Capacitor 470pF/100V, G1	39105-07	1
22	Capacitor 1nF/ 100V, G1	39105-10	1
23	Capacitor 47nF/ 250V, G1	39105-17	1
24	Capacitor 10nF/ 250V, G1	39105-14	1
25	Capacitor 100 nF/250V, G1	39105-18	1
26	Connection box	06030-23	1
27	Connecting cord, 32 A, 250 mm, red	07360-01	2
28	Connecting cord, 32 A, 250 mm, blue	07360-04	2
29	Connecting cord, 32 A, 750 mm, red	07362-01	2
30	Connecting cord, 32 A, 750 mm, yellow	07362-02	1
31	Short-circuit plug, white	06027-06	2
32	Connecting cord, 32 A, 750 mm, blue	07362-04	2
33	Headphone Adapter jack plug/2 x 4 mm plug	65974-01	1

Task

By using the Wheatstone bridge circuit, measure

1. ohmic resistances,
2. inductances and
3. capacitances.

Set-up and procedure

Set-up

Set up the experiment according to Fig. 1 and the respective schematic in Fig. 2–4. the digital frequency generator is connected in parallel to the measuring bridge.
For the measurement of resistances use the resistance of 1.0 kΩ (39104-19) as reference.
For the measurement of the inductances the variable resistance is needed to compensate for the different DC resistances in the coils. Use the induction coil (11006-01) as reference.
The Headphone is used instead of an Ampère-meter to balance the Wheatstone circuit.

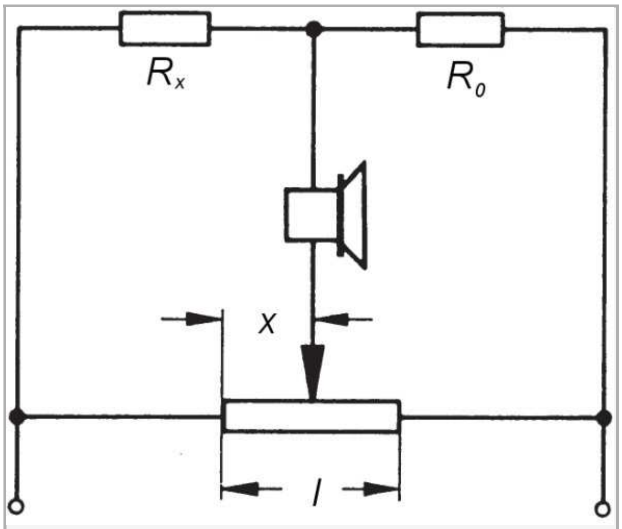

Fig. 2: Schematic circuit for task 1 (ohmic resistances).

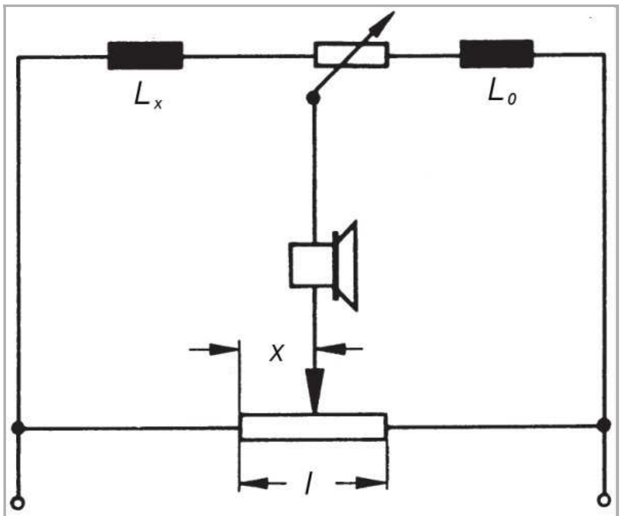

Fig. 3: Schematic circuit for task 2 (inductances).

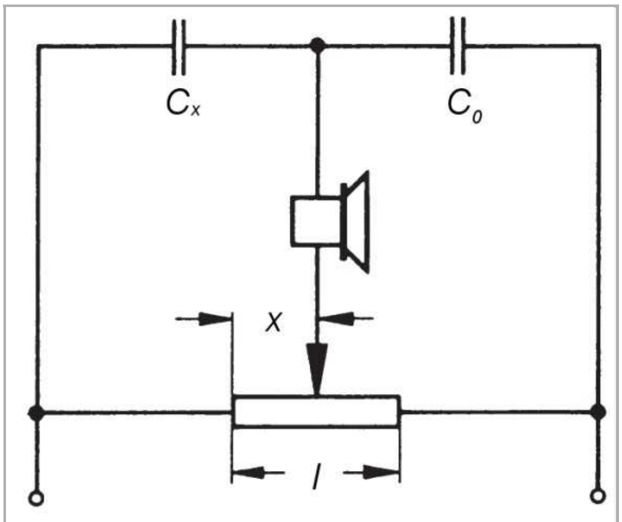

Fig. 4: Schematic circuit for task 3 (cpacitances).

Procedure

Choose a frequency on the frequency generator that is suited for the measurements, e.g. 400 Hz. Tune the amplitude to no more than 1 V before putting on the headphones.
In case of the measurement of inductances, try and find that resistance R_P for which the sound at a given slide position is lowest.

Move the slide into each direction until the sound minimizes or disappears entirely. Especially for the inductance measurement, only a range can be determined, for which the sound is lowest. Try to determine that range as carefully as possible and note down the medium point of that range as the distance with lowest amplitude.

When the sound amplitude becomes too low to hear, tune up the voltage output to a maximum of 20 V. Do not forget to tune it down as soon as you have found the balancing point and before changing any part of the circuit. Do not switch resistances/coils/capacitances while having such a high signal amplitude with headphones on – you are in danger of damaging your eardrums seriously!

Theory and evaluation

Theory

In principal an unknown resistance can be determined by measuring current and voltage across the resistance. The finite intrinsic resistances of the instruments would introduce significant errors. To avoid such errors the measurement has to be done current-free.
In a Wheatstone bridge circuit the unknown resistance R_x is connected two three known resistances (see Fig. 5) of which at least one is variable.
In this experiment R_1 and R_2 are adjusted in such manner that no current is flowing through the instrument G (alignment of the bridge) which means the voltage across G vanishes as well. In this case the voltages across R_x and R_1 are equal as well as across R_3 and R_2. Also, as no current is flowing through G, the same current is flowing through R_3 and R_x on the one hand (denoted as I_1) and through R_1 and R_2 on the other hand (I_2). This results into the following equations:

$$I_1 R_x = I_2 R_1 \text{ and } I_1 R_3 = I_2 R_2$$

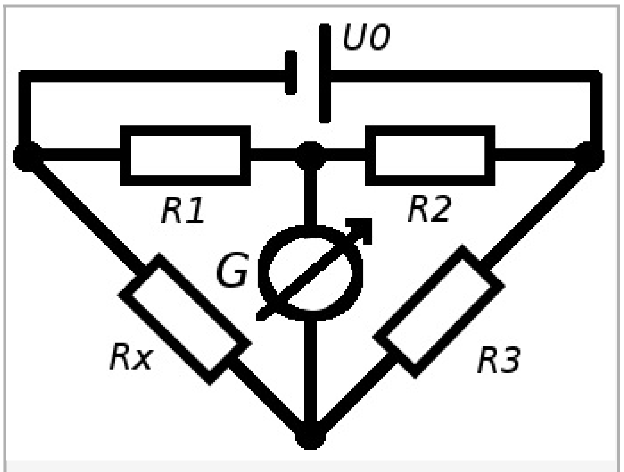

Fig. 5: Schematic circuit of the Wheatstone bridge

Division of these relations yields eq. (4) which computes the unknown resistance R_x.

$$R_x = R_3 \cdot \frac{R_1}{R_2}$$

The voltage at the source is unimportant and may even be time-dependent. In the general bridge construction we have impedances Z_3 and Z_x instead of the resistances R_3 and R_x. The current in the middle branch is zero when

$$\frac{R_1}{R_2} = \frac{Z_3}{Z_x} \tag{1}$$

In complex notation, the general form of the impedance Z is given by eq. (2) where $i = \sqrt{-1}$ is the complex number, $\omega = 2\pi\nu$ is the cyclic frequency and ν is the frequency of the applied voltage.

$$Z = R + i\left(\omega L - \frac{1}{\omega C}\right). \tag{2}$$

Substituting into (1) gives relation (3):

$$\frac{R_1}{R_2} = \frac{R_3 + i\left(\omega L_3 - \frac{1}{\omega C_3}\right)}{R_x + i\left(\omega L_x - \frac{1}{\omega C_x}\right)} \tag{3}$$

The real and imaginary parts must agree on both sides. Therefore we obtain two conditions. The first is the Amplitude condition eq. (4):

$$\frac{R_1}{R_2} = \frac{R_x R_3 + \left(\omega L_3 - \frac{1}{\omega C_3}\right)\left(\omega L_x - \frac{1}{\omega C_x}\right)}{R_x^2 + \left(\omega L_x - \frac{1}{\omega C_x}\right)}, \tag{4}$$

the second is the phase condition eq. (5):

$$R_3\left(\omega L_x - \frac{1}{\omega C_x}\right) = R_x\left(\omega L_3 - \frac{1}{\omega C_3}\right). \tag{5}$$

To fulfill (5), Z_3 and Z_x must consist of components that are alike.

Task 1

In the case of Z_3 and Z_x being ohmic resistances, (1) simplifies to

$$\frac{R_1}{R_2} = \frac{R_3}{R_x}. \tag{6}$$

The ratio R_1/R_2 is given by the length x and the total length l of the slide wire (Fig. 2). We thus obtain the unknown resistance with relation (7)

$$R_x = R_3 \cdot \frac{x}{l - x} \tag{7}$$

where $R_1 = l - x$ and $R_2 = x$.

Task 2

Besides an inductive ressitance, coils also have an ohmic reactance:

$$Z_x = R_x + i\omega L_x$$
$$Z_3 = R_3 + i\omega L_3$$

From (5) we first obtain relation (8):

$$\frac{R_3}{R_x} = \frac{L_3}{L_x} \tag{8}$$

In order to be able to fulfill this condition, we have to connect an additional resistance into the bridge branch with the coils (see Fig. 3).R_3 and R_x is then the sum of the additional resistance and the ohmic resistance of the coil in the corresponding branch. Substituting (8) into (5) we obtain

$$\frac{R_1}{R_2} = \frac{L_3}{L_x}$$

so that relation (9) holds:

$$L_x = L_3 \frac{x}{l - x} \tag{9}$$

The inductance of the reference coil (Art. No. 11006-01) is determined from the dimensions:

$$L_3 = 2.1 \cdot 10^{-6} \cdot N^2 \cdot r \cdot (\frac{r}{l})^{3/4}$$

where L is in H, radius r and length l are in m, and N is the number of turns. For the given coil one obtains $L_3 = 0.8$ mH.

Task 3

For capacitances we obtain from relation (5)

$$\frac{R_1}{R_2} = \frac{C_4}{C_3}$$

and thus

$$C_x = C_3 \frac{l - x}{x} \qquad (10)$$

Evaluation and results

In the following the evaluation of the obtained values is described with the help of example values. Your results may vary from those presented here.
For Task 1, reference resistance 1.0 kΩ was used. In Tab. 1 the measured values are shown and the calculated resistances are compared to the expected values. Most values compare quite well, only the values for Art. No. 39104-21 deviate from the expectation.

Tab. 1: Measured and calculated values for Task 1.

Resistor (Art. No.)	x / mm	R_x / Ω (calc)	R_x / Ω (theo)
39104-13	244	323	330
39104-15	317	464	470
39104-17	402	672	680
39104-19	502	1008	1000
39104-21	616	1604	1500
39104-23	688	2205	2200
39104-25	768	3310	3300

In Tab. 2 the measured values of Task 2 are shown and the calculated inductances are compared to the expected values. The values compare quite well.

Tab. 2: Measured and calculated values for Task 2. R_P denotes the additional resistance that compensates the coil's DC resistance.

Inductance (Art. No.)	x / mm	R_P / Ω	L_x / mH (calc)	L_x / mH (theo)
06513-01	620	1	1.3	2
06514-01	920	2	9.2	9
06515-01	975	10	31.2	35
06522-01	950	5	15.2	15

For Task 3 the capacitance of 1 nF (39105-10) had been used as reference. The measured values are shown in Tab. 3 as well as calculated and expected capacitances. The experimental values compare very well with theoretical values.

Tab. 3: Measured and calculated values for Task 3.

Capacitance (Art. No.)	x / mm	C_x / nF (calc)	C_x / nF (theo)
39105-04	910	0.10	0.1
39105-07	680	0.47	0.47
39105-14	100	9.00	10
39105-18	5	99.00	100